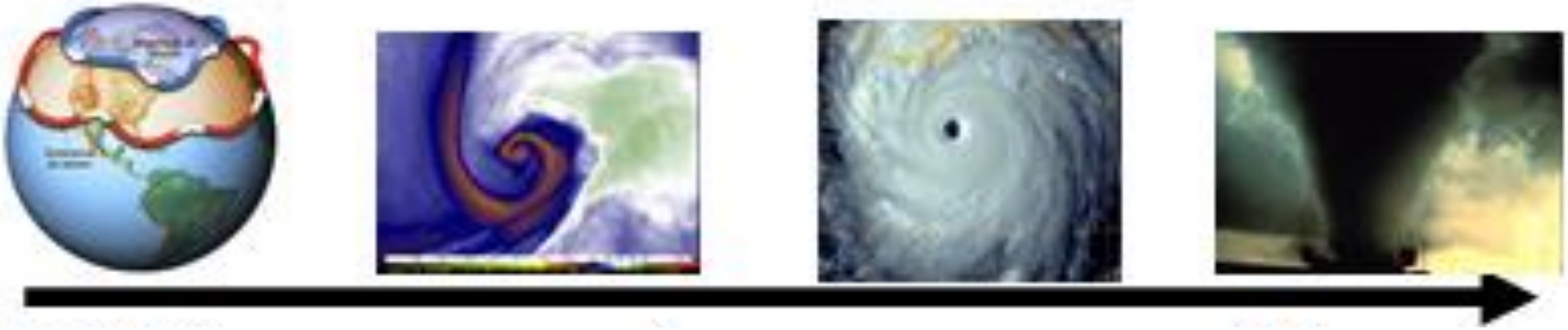
P1: Vortices in the atmosphere

http://weatherclimatelab.mit.edu/projects/weather-and-extremes/observation-data



jet stream

blizzard

hurricane

tornado

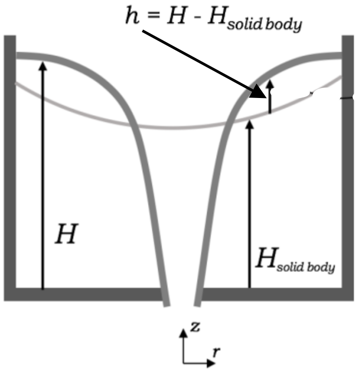
Summary of theory

Inertial Frame

$$\boxed{V_\theta = v_\theta + \Omega r}$$

Rotating Frame

$$\frac{V_\theta^2}{r} = g\frac{\partial H}{\partial r}$$



$$\frac{v_\theta^2}{r} + 2\Omega v_\theta = g\frac{\partial h}{\partial r}$$

$$\boxed{Ro = \frac{|v_\theta^2/r|}{|2\Omega v_\theta|} = \frac{|v_\theta|}{2\Omega r}}$$

$$R_{timescales} = \frac{2\pi/\Omega}{2\pi r/v_\theta} = \frac{v_\theta}{\Omega r} = 2 \times R_o$$

Three limits:

$R_0 \ll 1$

$$2\Omega v_\theta = g\frac{\partial h}{\partial r}$$

Geostrophic balance

$R_0 \sim 1$

$$\frac{v_\theta^2}{r} + 2\Omega v_\theta = g\frac{\partial h}{\partial r}$$

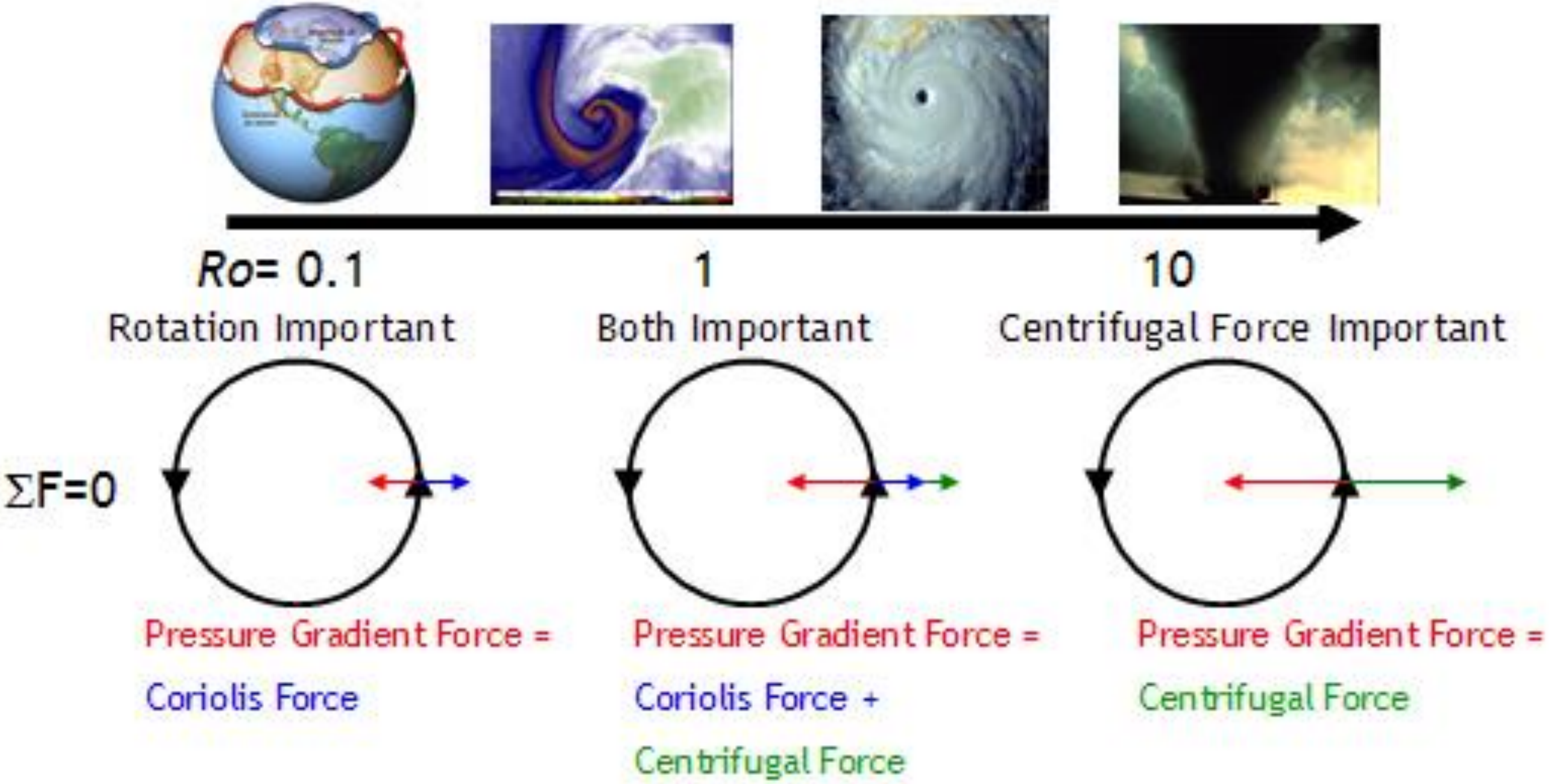
Gradient wind balance

$R_0 \gg 1$

$$\frac{v_\theta^2}{r} = g\frac{\partial h}{\partial r}$$

Cyclostrophic balance

Atmospheric vortices: balance of forces



Ro= 0.1

Rotation Important

1

Both Important

10

Centrifugal Force Important

ΣF=0

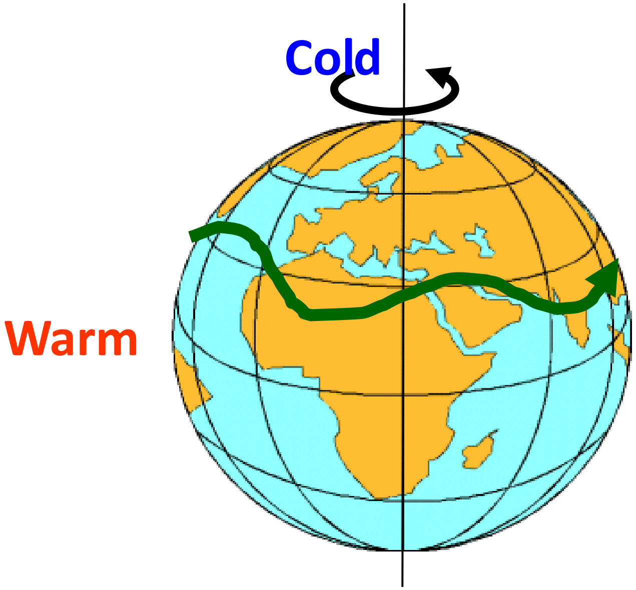
Pressure Gradient Force =
Coriolis Force

Pressure Gradient Force =
Coriolis Force +
Centrifugal Force

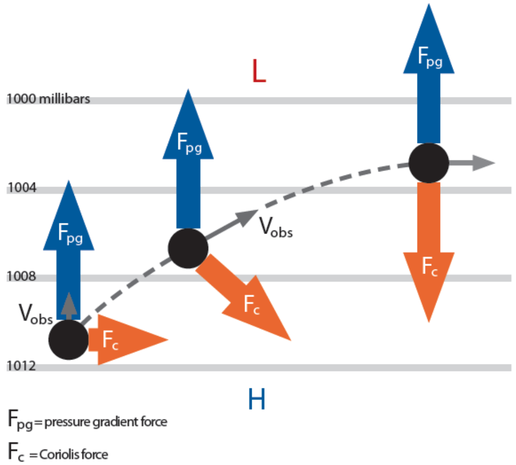
Pressure Gradient Force =
Centrifugal Force

Jet stream in geostrophic balance

The Equator-to-pole temperature difference induces a meridional (north-south) pressure gradient, with a **Low** pressure over the **Cold** Pole



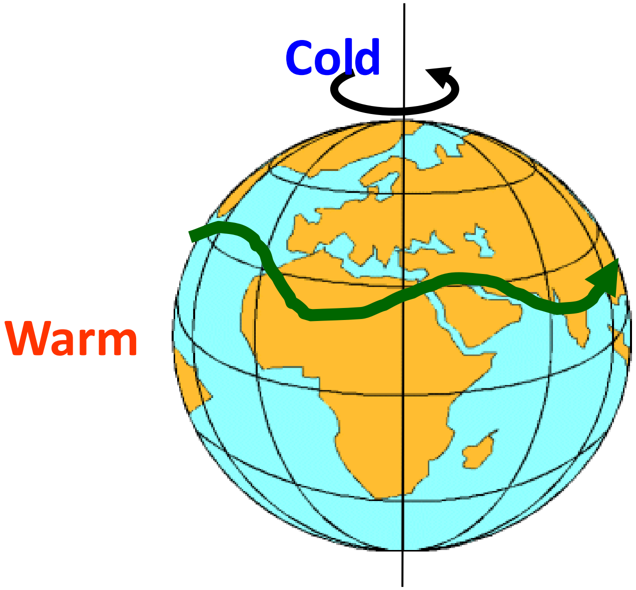
The jet stream is in therefore in *geostrophic balance:* the pressure gradient force is balanced by the Coriolis force



At altitude, friction with the Earth lessens and the pressure gradient and the Coriolis forces balance out.

Jet stream in geostrophic balance

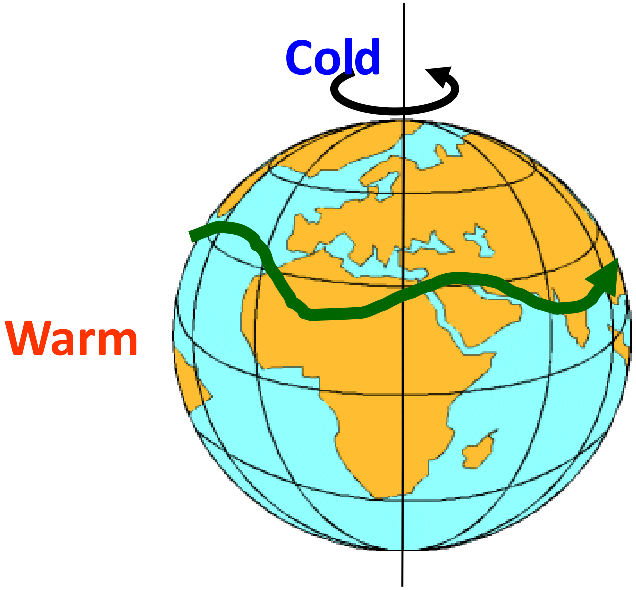
- Because of the N-S temperature difference, pressure gradient force is increasing with height.

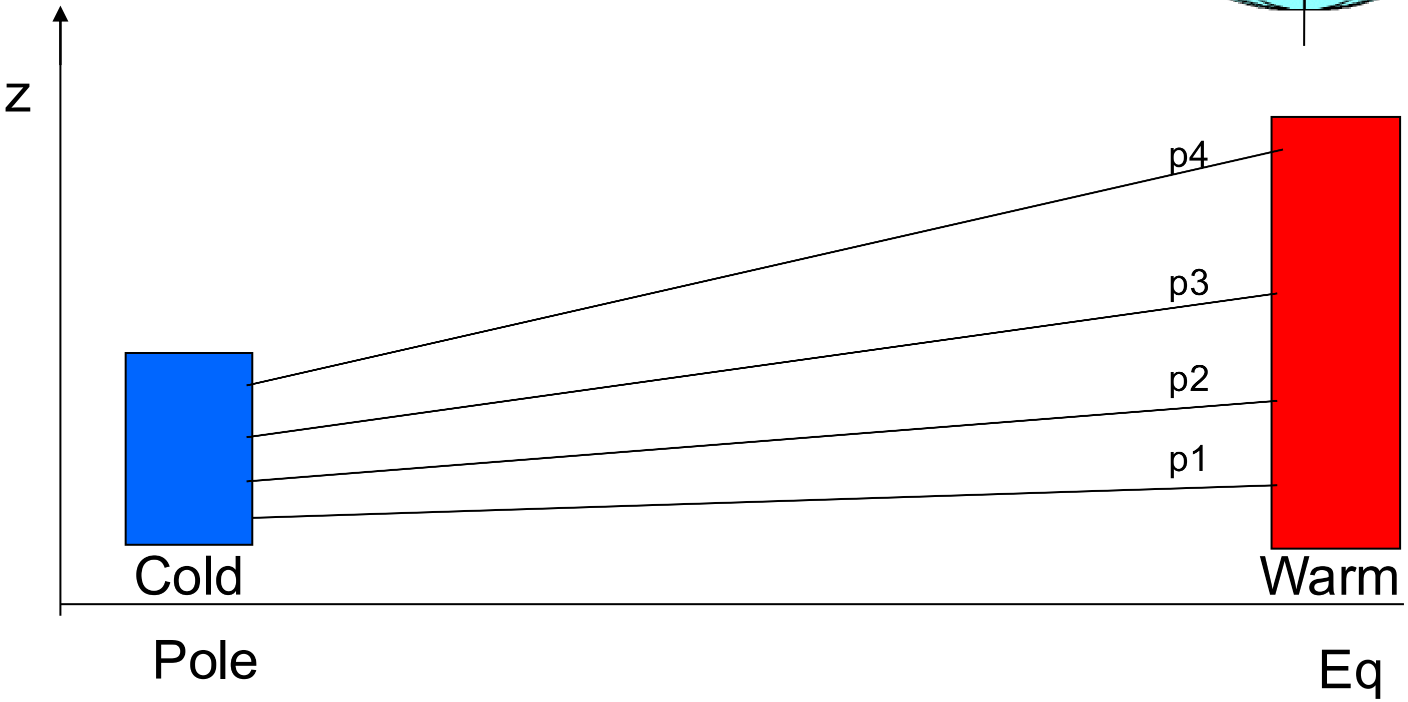


Jet stream in geostrophic balance

- Because of the N-S temperature difference, pressure gradient force is increasing with height.
- The geostrophic wind is therefore also increasing with height (reaching a maximum at the tropopause)

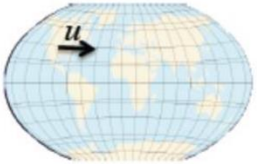
→ *Thermal wind balance!*

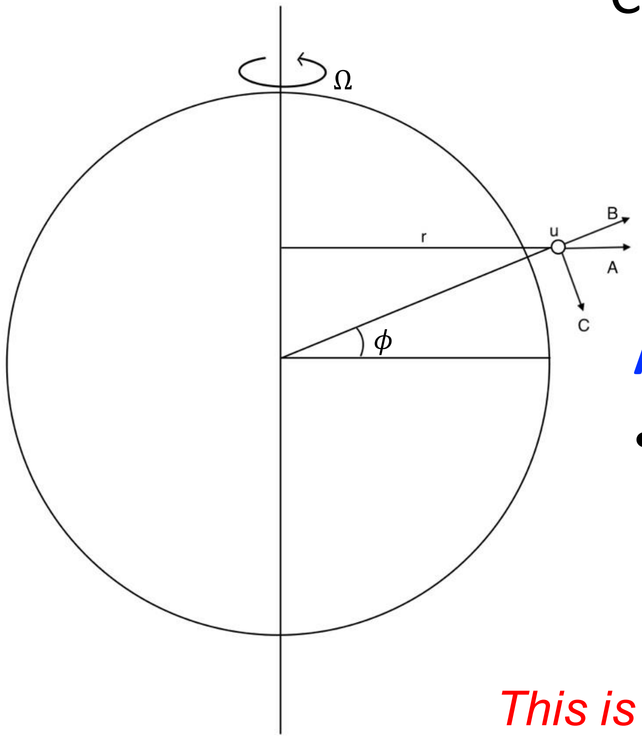




Coriolis Force on a sphere

Consider a parcel moving from West to East with velocity U





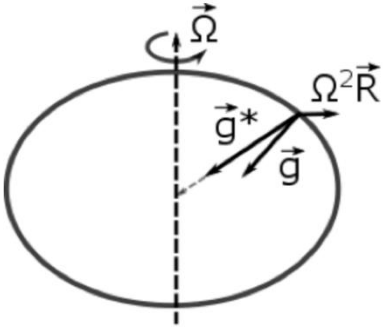
Centrifugal Acceleration:

$$A = \frac{V^2}{r} = \frac{(U + \Omega r)^2}{r} = \underbrace{\frac{U^2}{r}}_{\text{small}} + 2\Omega U + \underbrace{\Omega^2 r}_{\text{Included in g}}$$

A can be resolved into two components, B and C:

- B is perpendicular to the Earth's Surface and changes the weight of the ring slightly

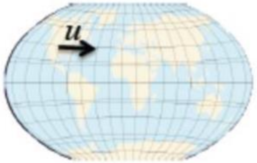
This is why Earth's surface is not actually a perfect sphere!

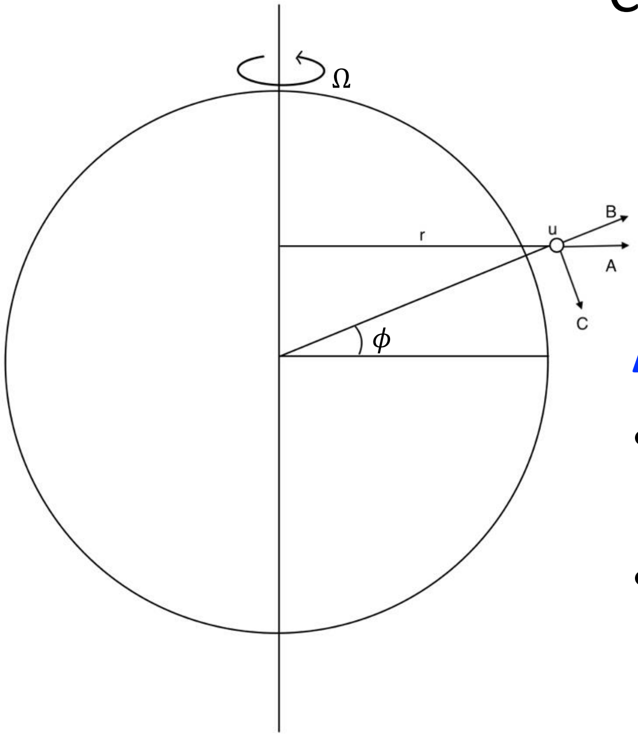


In reality, the geopotential surfaces depart only very slightly from a sphere, being ~11 km higher at the equator than at the pole

Coriolis Force on a sphere

Consider a parcel moving from West to East with velocity U





Centrifugal Acceleration:

$$A = \frac{V^2}{r} = \frac{(U + \Omega r)^2}{r} = \underbrace{\frac{U^2}{r}}_{\text{small}} + 2\Omega U + \underbrace{\Omega^2 r}_{\text{Included in g}}$$

A can be resolved into two components, B and C:

- B is perpendicular to the Earth’s Surface and changes the weight of the ring slightly
- C is parallel to the Earth’s Surface:

$$\boxed{C = 2\Omega \sin\phi \, u = fu}$$

is the Coriolis acceleration, and $\mathrm{f} = 2\Omega \sin\phi$ is the Coriolis parameter

The direction of the acceleration depends on the sign of u! (zero if at rest)

Coriolis Force on a sphere

Facts about Coriolis force:

$$\boxed{F_c = 2\Omega \sin\phi \, u = fu}$$

- The Coriolis force causes objects to turn to the right of their direction of motion in the Northern hemisphere, and to the left in the Southern hemisphere!
- It can affect the direction of motion but not the speed!
- The magnitude of Coriolis force depends on:

(1) the rotation of the Earth

(2) the speed of the moving object (strongest for fast moving objects and zero at rest)

(3) its latitudinal location (is zero at the equator and maximum at the poles)

- The Coriolis force is most important for air movement over large distances and becomes insignificant at small scales

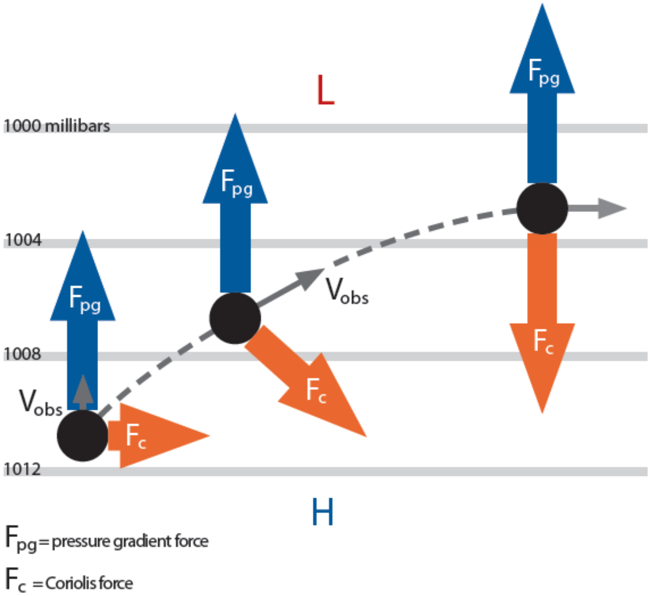
Coriolis force is very important process and shapes weather patterns

Geostrophic balance on the sphere

If Coriolis and PGF are in balance-

$$\boxed{\begin{aligned} u &= -\frac{1}{\rho f}\frac{\partial p}{\partial y} \\ v &= \frac{1}{\rho f}\frac{\partial p}{\partial x} \end{aligned}}$$

where $f = 2\Omega\ \sin\phi$ is the Coriolis parameter



At altitude, friction with the Earth lessens and the pressure gradient and the Coriolis forces balance out.

This is the geostrophic wind resulting from the balance between the PGF and the Coriolis force

Geostrophic balance with friction

At the surface, friction plays an important role:

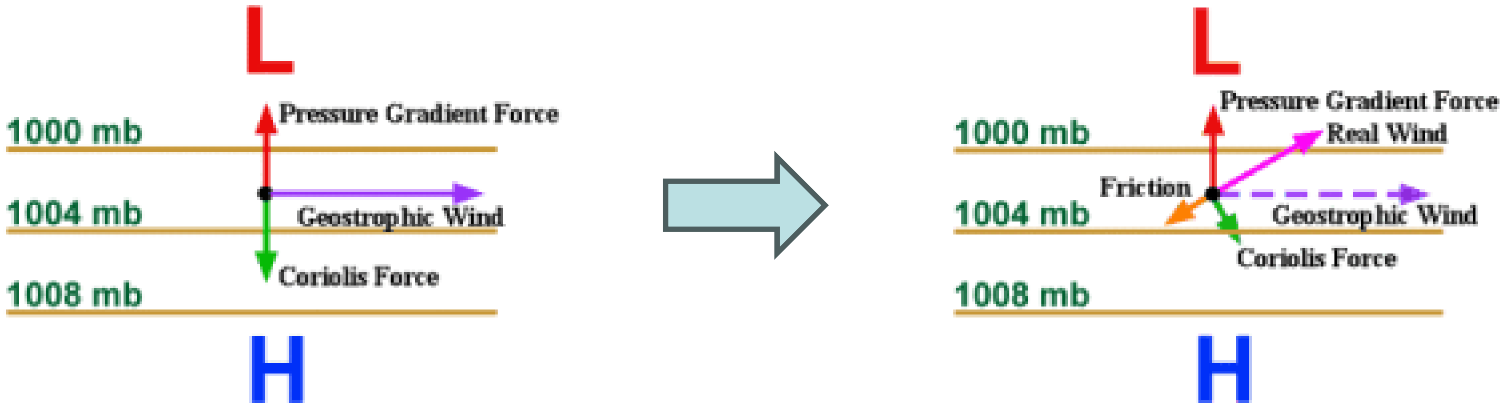


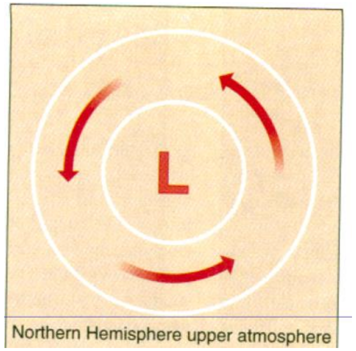
Figure from: http://ww2010.atmos.uiuc.edu/(Gh)/guides/mtr/fw/fric.rxml

Surface friction slows down the parcel → PGF "wins"

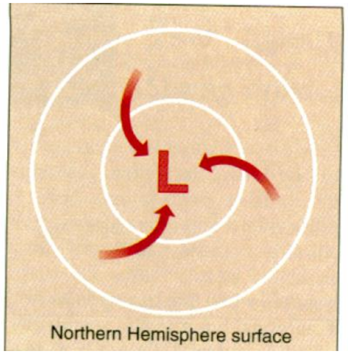
The flow is generally away from the high and towards the low

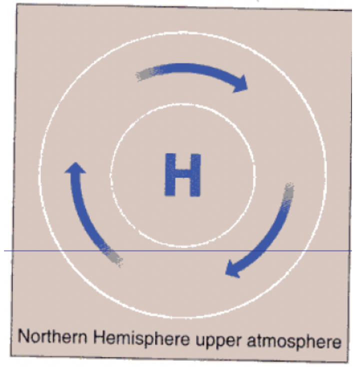
Geostrophic balance with friction

At the surface, friction plays an important role:

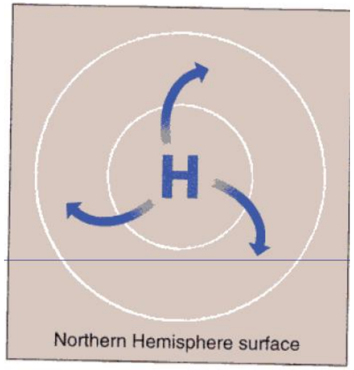


cyclones





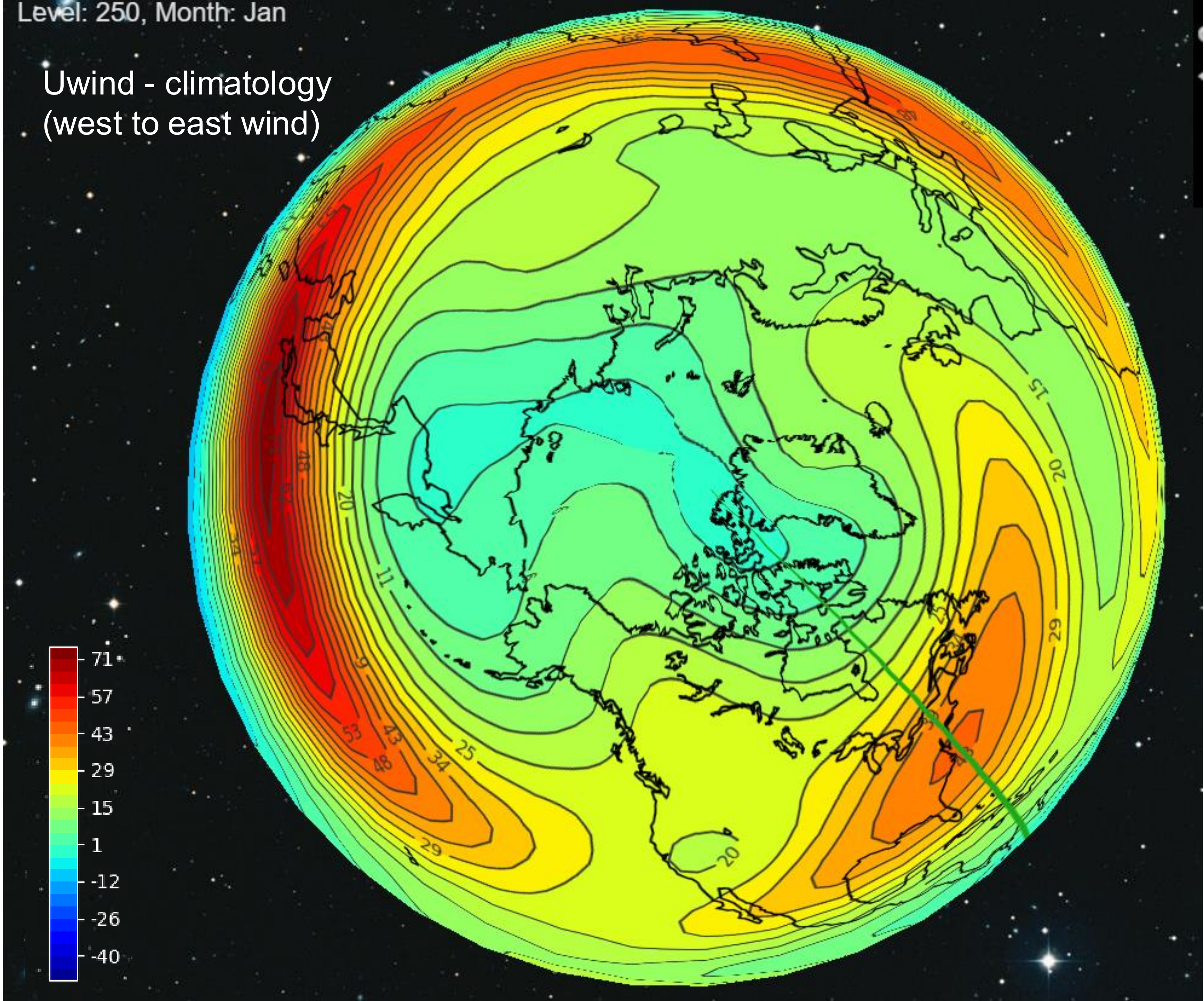
Anticyclones



The flow is slightly away from the high and towards the low

Is the jet stream in geostrophic balance?

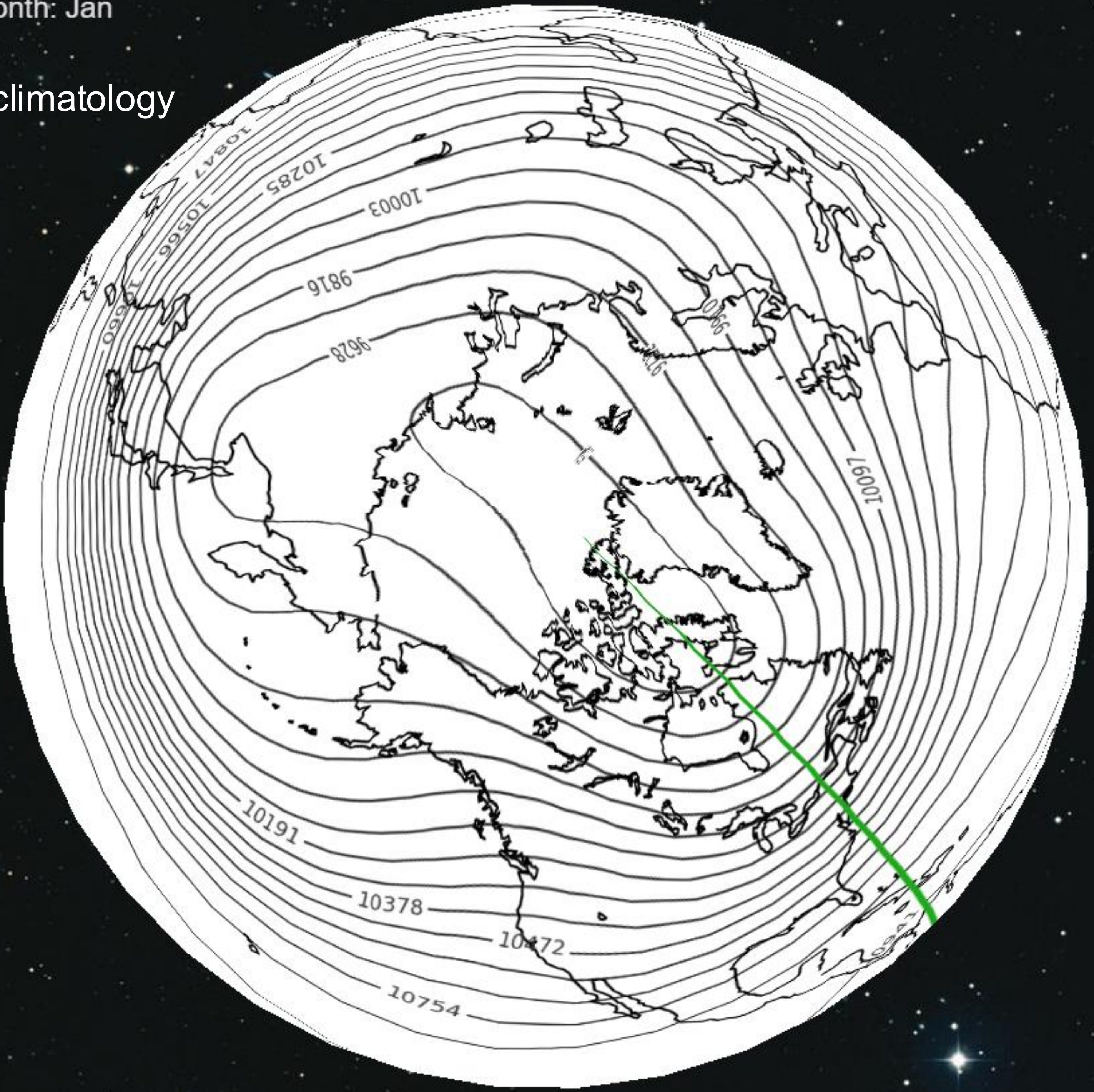
Use the EsGlobe January climatology to verify it!



Uwind - climatology
(west to east wind)

Level: 250, Month: Jan

Height climatology



Jet stream in geostrophic balance ?

$$fu = -\frac{1}{\rho}\frac{\partial p}{\partial y} \xrightarrow{\frac{\partial p}{\partial z} = -\rho g} \boxed{u = -\frac{g}{f}\frac{\partial h}{\partial y}}$$

Coriolis Force

Pressure Gradient Force

$$\boxed{u \cong -\frac{g}{f}\frac{\Delta h}{\Delta y}}$$

Put numbers in…

$\Delta h \cong ?$

$\Delta y \cong ?$

$$\boxed{u \cong 30 - 60\ \frac{m}{sec}}$$

where $f = 2\Omega \sin\phi$ and $\Omega \cong 7 \cdot 10^{-5}$

Extratropical weather systems

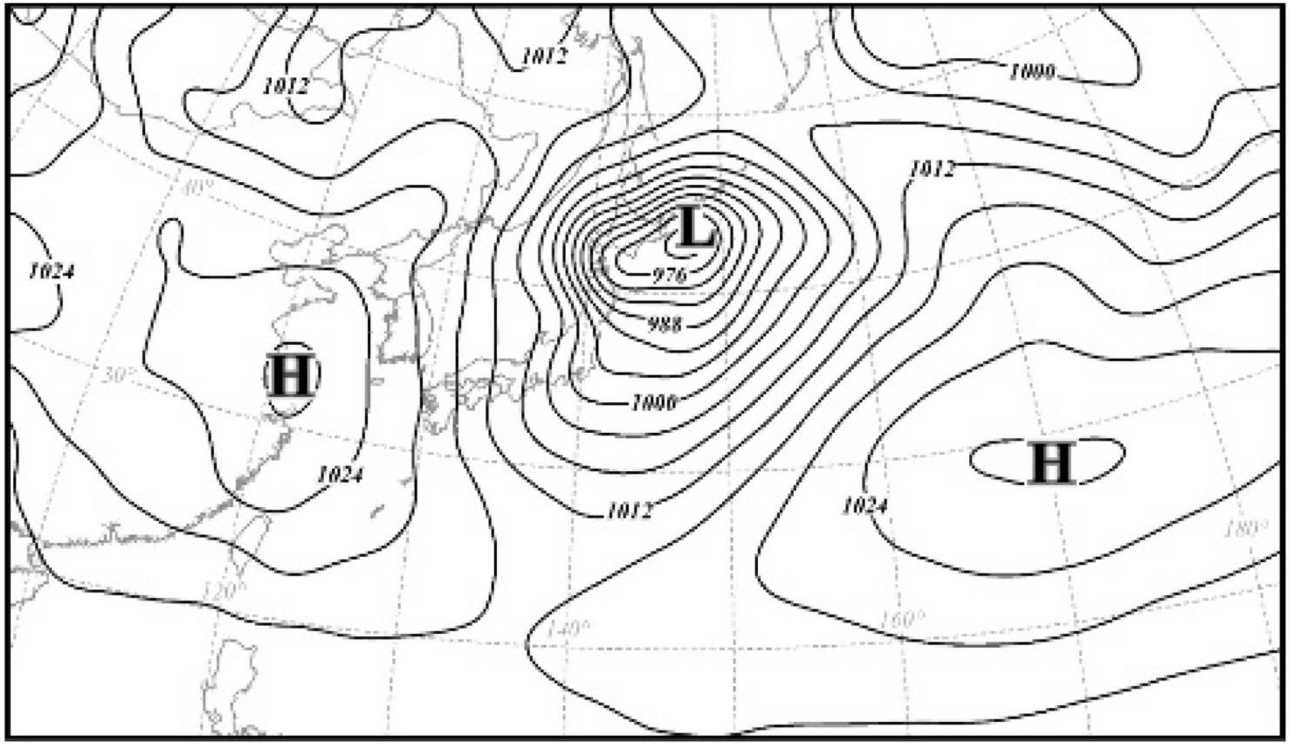


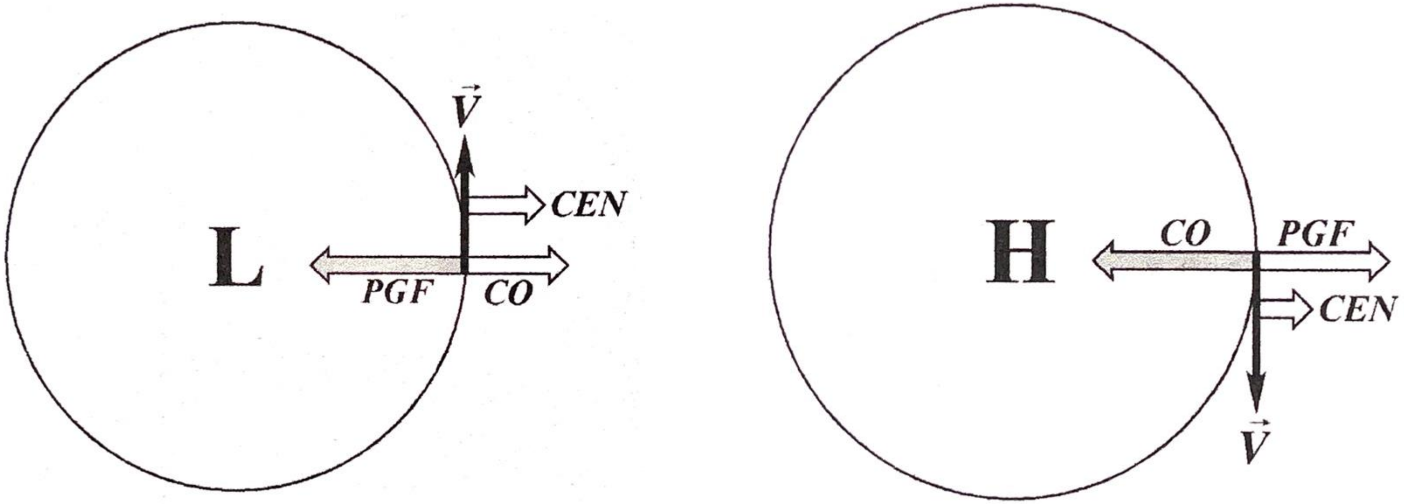
Figure 4.20 Sea-lavel pressure analysis for 0000 UTC 23 February 2004. Solid lines are isobars labeled in hPa and contoured every 4 hPa. Capital L and H represent centers of sea-level low- and high-pressure systems, respectively. Note the tight pressure gradient around the low and the much weaker pressure gradient around the highs

Figure taken from "Mid-Latitude Atmospheric Dynamics: A First Course", book by Jonathan E. Martin

Why are anticyclones (H) weaker and spatially larger than cyclones (L)?

The gradient wind balance-

Balance of forces for cyclones (L) and anticyclones (H)



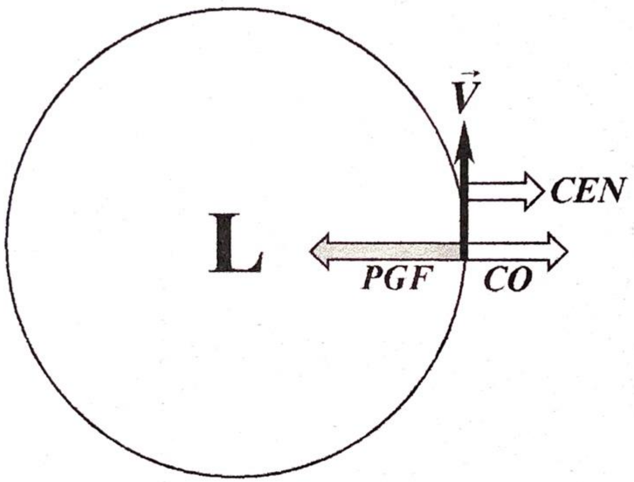
$$\frac{v^2}{r} + fv = g\frac{\partial h}{\partial r}$$

$$\boxed{\frac{v^2}{r} + fv - g\frac{\partial h}{\partial r} = 0}$$

$$\boxed{\frac{v^2}{r} - f|v| + g|\frac{\partial h}{\partial r}| = 0}$$

The gradient wind balance-

Balance of forces for cyclones (L):



$$\frac{v^2}{r} + fv - g\frac{\partial h}{\partial r} = 0$$

Note that $f v_g = g\frac{\partial h}{\partial r}$

$$\Rightarrow \quad \boldsymbol{f(v_g - v) = \frac{v^2}{r} > 0}$$

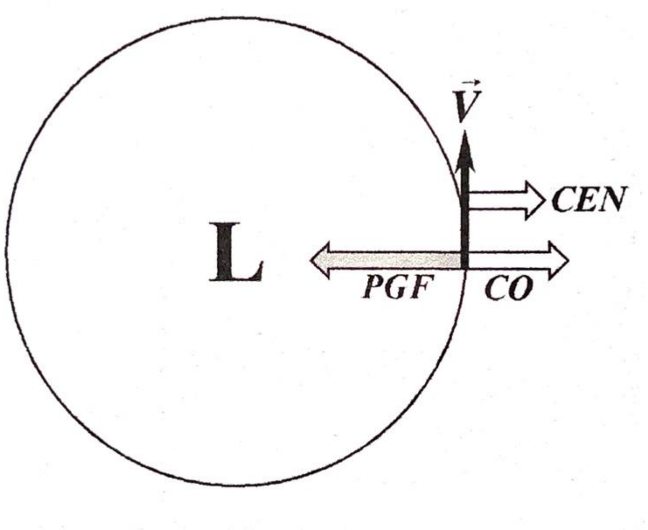
$$\Rightarrow \quad \boxed{v < v_g}$$

Subgeostrophic

The cyclonic wind is always weaker than the geostropic wind!

The gradient wind balance-

Balance of forces for cyclones (L):



$$\frac{v^2}{r} + fv - g\frac{\partial h}{\partial r} = 0$$

$$v = \frac{-fr}{2} \pm \sqrt{\frac{f^2r^2}{4} + rg\frac{\partial h}{\partial r}}$$

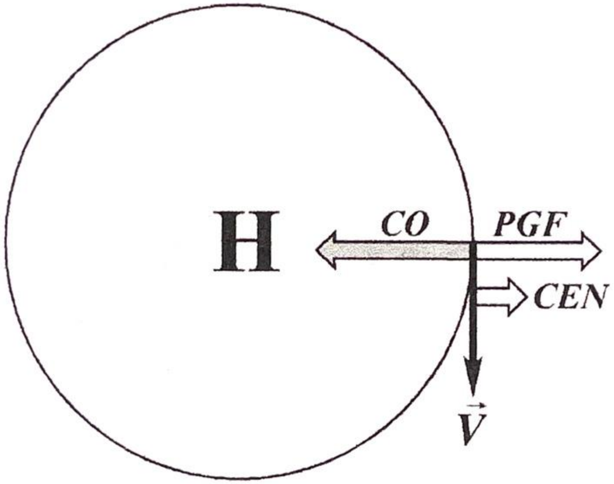
$$v_1 = -\frac{fr}{2} + \sqrt{\frac{f^2r^2}{4} + rg\frac{\partial h}{\partial r}} > 0$$

$$v_2 = -\frac{fr}{2} - \xcancel{\sqrt{\frac{f^2r^2}{4} + rg\frac{\partial h}{\partial r}}} < 0$$

$$\boxed{v = -\frac{fr}{2} + \sqrt{\frac{f^2r^2}{4} + rg\frac{\partial h}{\partial r}}}$$

The gradient wind balance-

Balance of forces for anticyclones (H):



$$\frac{v^2}{r} + fv - g\frac{\partial h}{\partial r} = 0$$

Note that $fv_g = g\frac{\partial h}{\partial r}$

$$\Rightarrow \quad \boldsymbol{f(|v| - |v_g|) = \frac{v^2}{r} > 0}$$

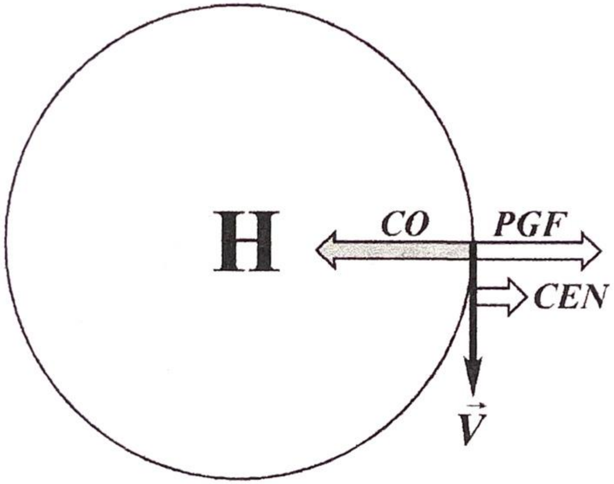
$$\Rightarrow \quad \boxed{|v| > |v_g|}$$

Supergeostrophic

The anticyclonic wind is always stronger than the geostropic wind!

The gradient wind balance-

Balance of forces for anticyclones (H):



$$\frac{v^2}{r} + fv - g\frac{\partial h}{\partial r} = 0$$

$$v = -\frac{fr}{2} \pm \sqrt{\frac{f^2r^2}{4} - rg|\frac{\partial h}{\partial r}|}$$

Solutions only if: $\frac{f^2r^2}{4} - rg|\frac{\partial h}{\partial r}| > 0$ → $\boxed{g|\frac{\partial h}{\partial r}| < \frac{f^2r}{4}}$

$$\boxed{v = \frac{-fr}{2} - \sqrt{\frac{f^2r^2}{4} - rg|\frac{\partial h}{\partial r}|}}$$

The pressure gradient of an anticyclone is bounded!

Also, $|\frac{\partial h}{\partial r}| \rightarrow 0$ when $r \rightarrow 0$

Let's check if this simple theory works for real H and L!

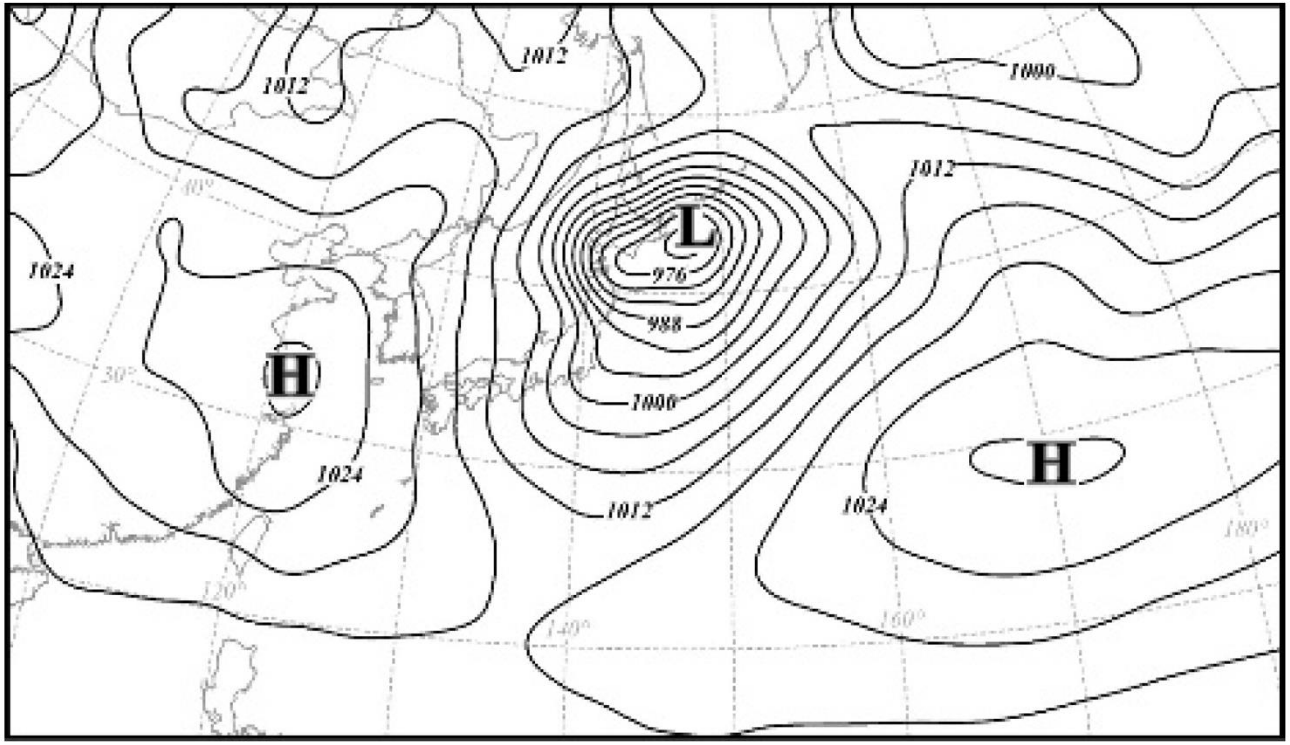


Figure 4.20 Sea-lavel pressure analysis for 0000 UTC 23 February 2004. Solid lines are isobars labeled in hPa and contoured every 4 hPa. Capital L and H represent centers of sea-level low- and high-pressure systems, respectively. Note the tight pressure gradient around the low and the much weaker pressure gradient around the highs

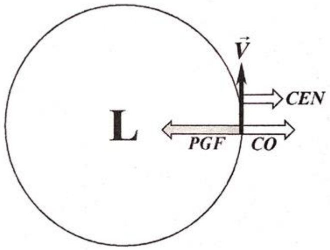
Figure from "Mid-Latitude Atmospheric Dynamics: A First Course", book by Jonathan E. Martin

Note: the fact that $v < v_g$ for cyclones and $|v| > |v_g|$ for anticyclones ***does not*** mean that anticyclones velocities are larger. ***This is true only for the same pressure gradient!***

Note also that-

For cyclones, we had

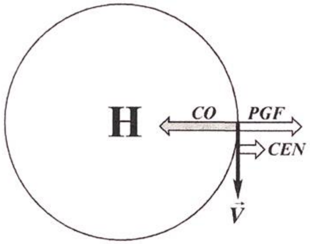
$$\frac{v^2}{r} + fv - fv_g = 0$$



$$\Rightarrow \quad \frac{v^2}{fr} = v_g - v \qquad \Rightarrow \quad \frac{v_g}{v} - 1 = \frac{v}{fr} \equiv R_0$$

For cyclones: $\boxed{\frac{v_g}{v} = R_0 + 1}$ $\Rightarrow$ $\boxed{v < v_g}$

Similarly, for anticyclones we get $\frac{v^2}{r} - f|v| + f|v_g| = 0$



For anticyclones: $\boxed{\frac{|v_g|}{|v|} = 1 - R_0}$ $\Rightarrow$ $\boxed{|v| > |v_g|}$

Let’s check if this simple theory works for real H and L!

We will use the Synoptic Laboratory website (Lodo Illari) to plot v, v_g for real atmospheric data.

- Check February 16th 2024, around Washington (WA state) for a nice example.
- Go to [*http://synoptic.mit.edu/custom-plots/anyscalarwind/*](http://synoptic.mit.edu/custom-plots/anyscalarwind/)
- Set:

1. In the “Scaler” field, change “tmpc” to “hght”
2. Set the day to “16” instead of today, and year to 2024.
3. In the “Wind-skip” option, change to yes (to reduce the number if arrows)
4. In the GAREA option, change “usnps” to “WA--”.

→ This will produce a map with wind barbs and the 500mb geopotential height

- Now repeat, but in the “Wind” option change “observed” to “Geostrophic”

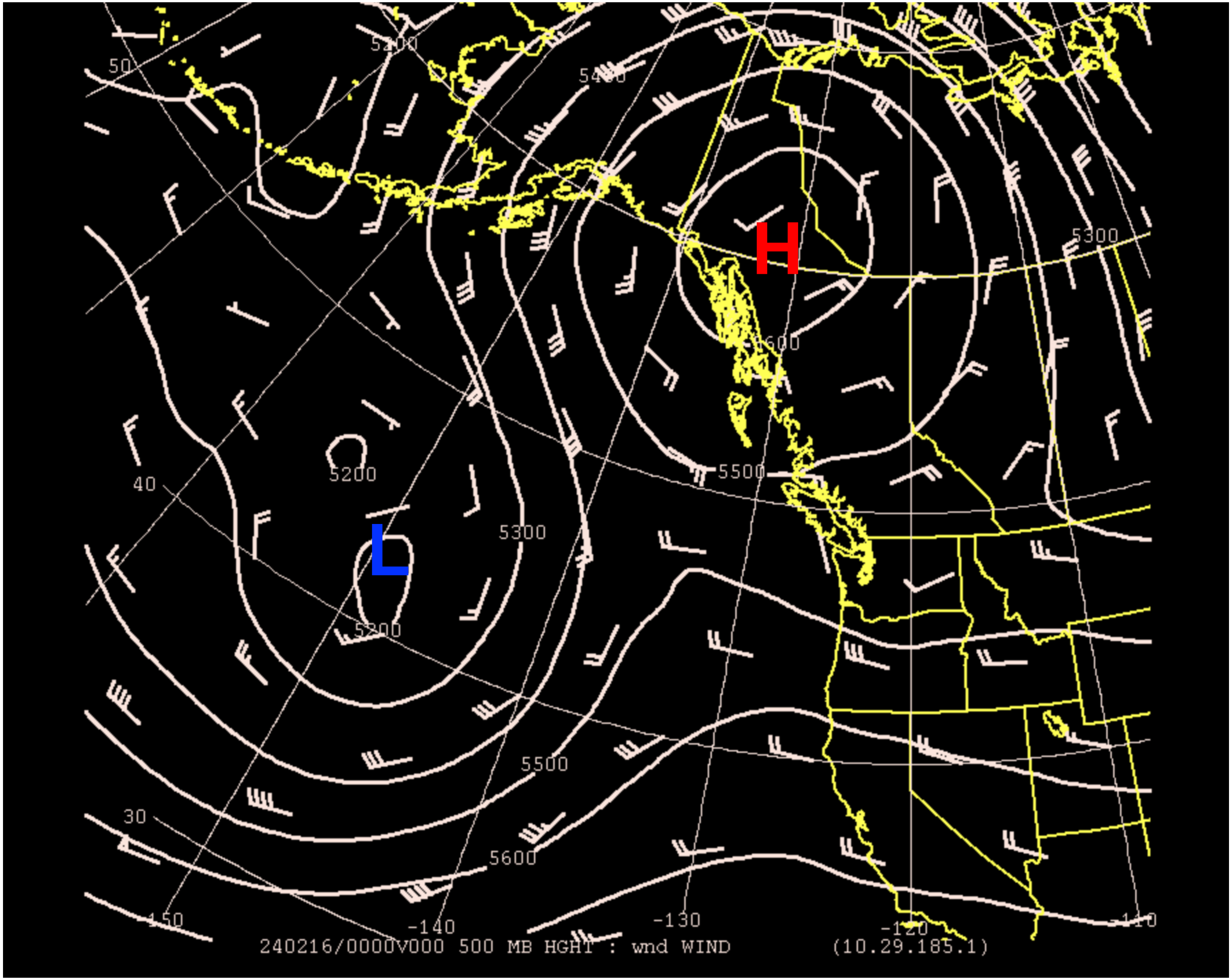
- Is the actual v smaller or larger than the geostrophic velocity in the L/H region?

- Can you estimate the Rossby number?

cyclones $\boxed{R_0 = \frac{v_g}{v} - 1}$

anticyclones $\boxed{R_0 = 1 - \frac{v_g}{v}}$

Total wind


H
L
5200
5400
5300
5600
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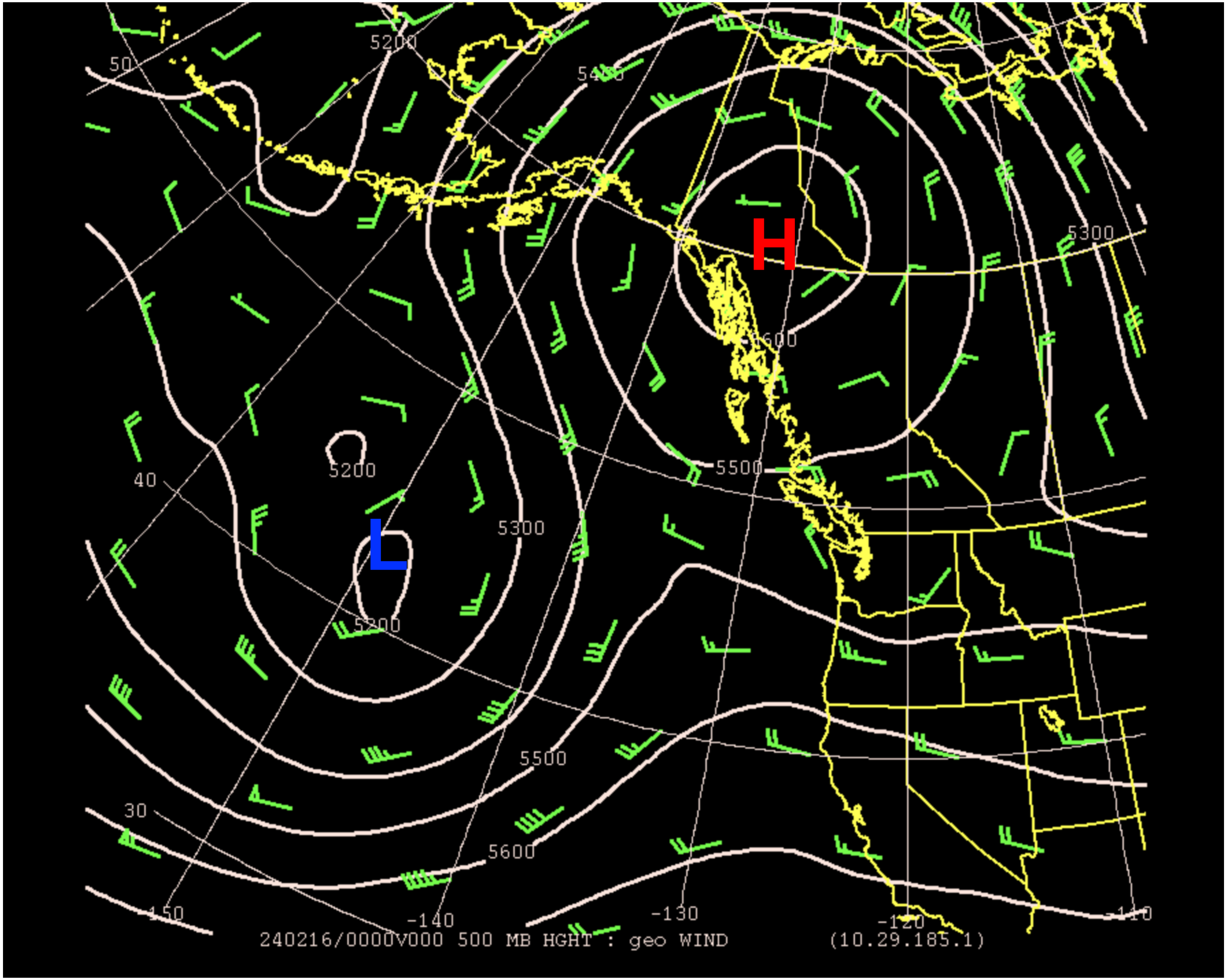
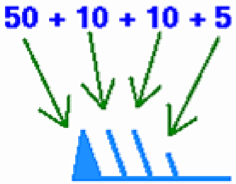
5300
5200
5500
5600
50
40
30
-150
-140
-130
-120
-110
240216/0000V000 500 MB HGHT : wnd WIND
(10.29.185.1)

Geostrophic wind

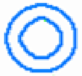




Wind blowing from the west at 75 knots

Wind blowing from the northeast at 25 knots

Wind blowing from the south at 5 knots



Calm winds

Hurricanes-

Next class!

EsGlobe uses a **global** dataset:

Winds from the **GFS - Global Forecast Model** (NCEP)

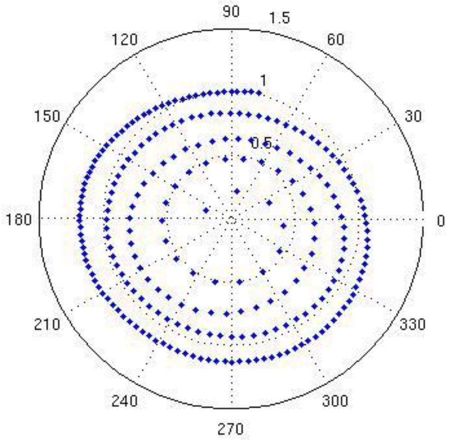
lat, lon grid with a resolution of ¼ of degree = ~ 25km

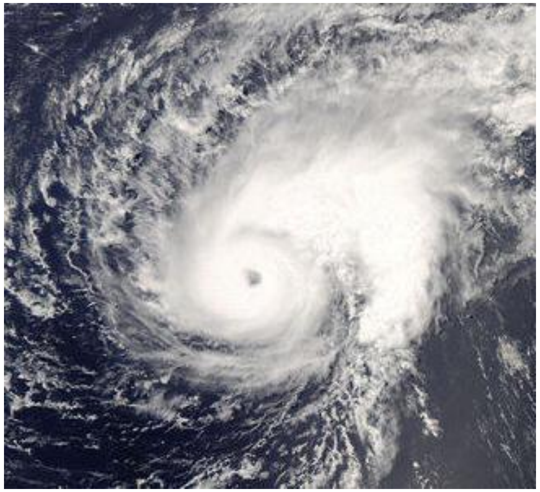
Not enough resolution to represent well an hurricane, which has a radius of few hundreds km

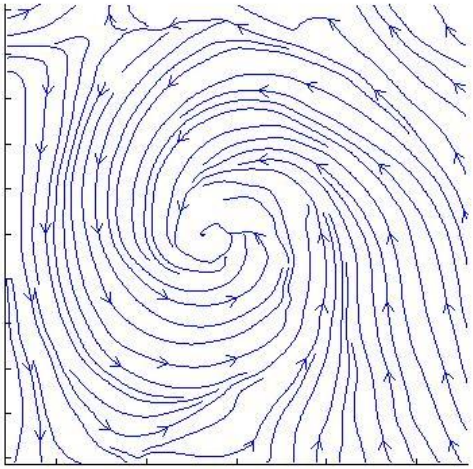
To study the balance of forces in a hurricane we are using a special dataset: surface wind data from the "**scatterometer**" instrument

See scatterometer_instructions

Hurricane flow and the balanced vortex experiment






Streamlines for Bertha 20080711