

P3: Heat and Moisture Transport

The general circulation



12.307- project 3 (data class 1)

Heat transport in the atmosphere

First class- The Hadley circulation

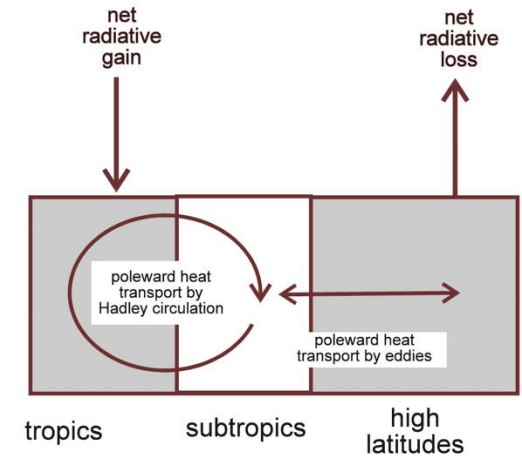
- Identify the Hadley circulation (EsGlobe)
- N-S heat transport in the tropics; Two-layer model
- Moisture transport in the tropics

Second class- Energy transport

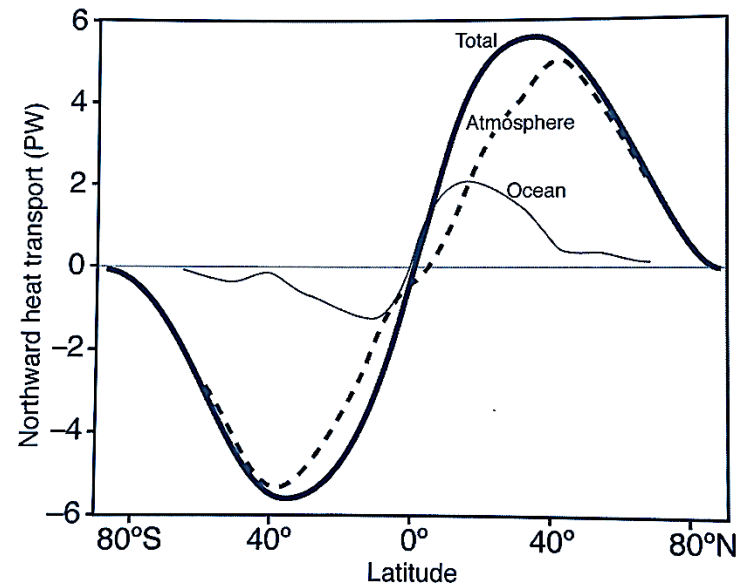
- Energy transport in the tropics - review calculations
- Energy transport in the Extra-tropics: eddy regime

Earth's meridional heat transport

- In the **tropics**, heat transport is mainly by the **Hadley** cell
- In the **extratropics**, heat transport is by the “**eddies**”



The atmosphere transfers most of the heat, but the ocean also contributes in the tropics



Hadley Cell

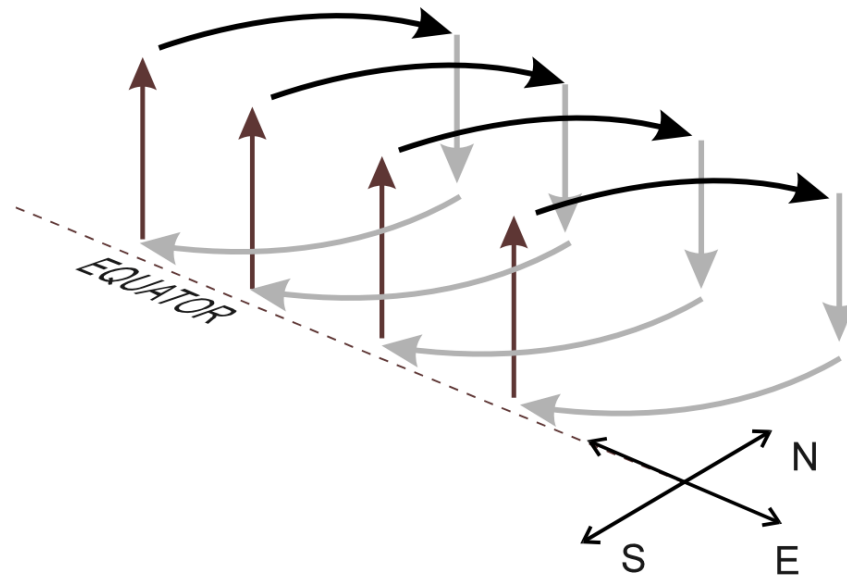


Figure 8: Schematic of the Hadley circulation (showing only the N Hem part of the circulation; there is a mirror image circulation south of the equator).

Zonally symmetric cell, meridional (North-South) circulation

Hadley Cell

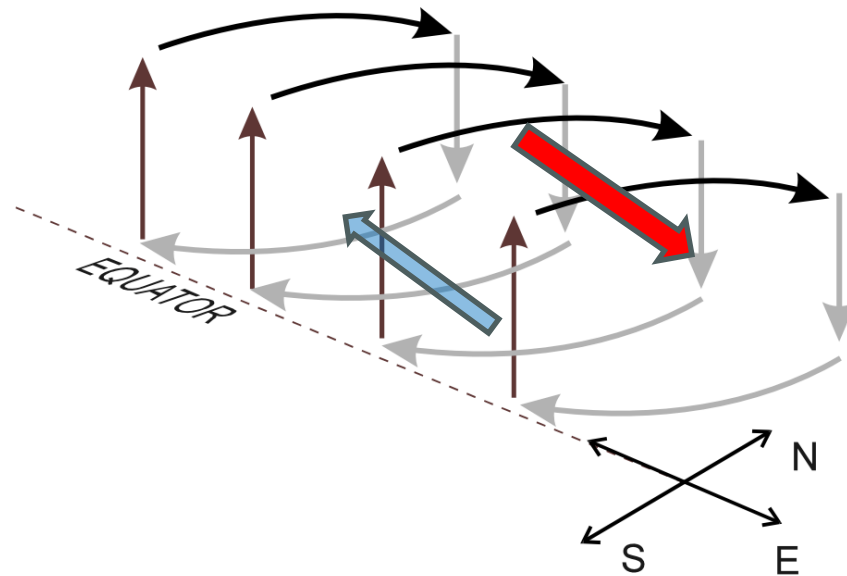
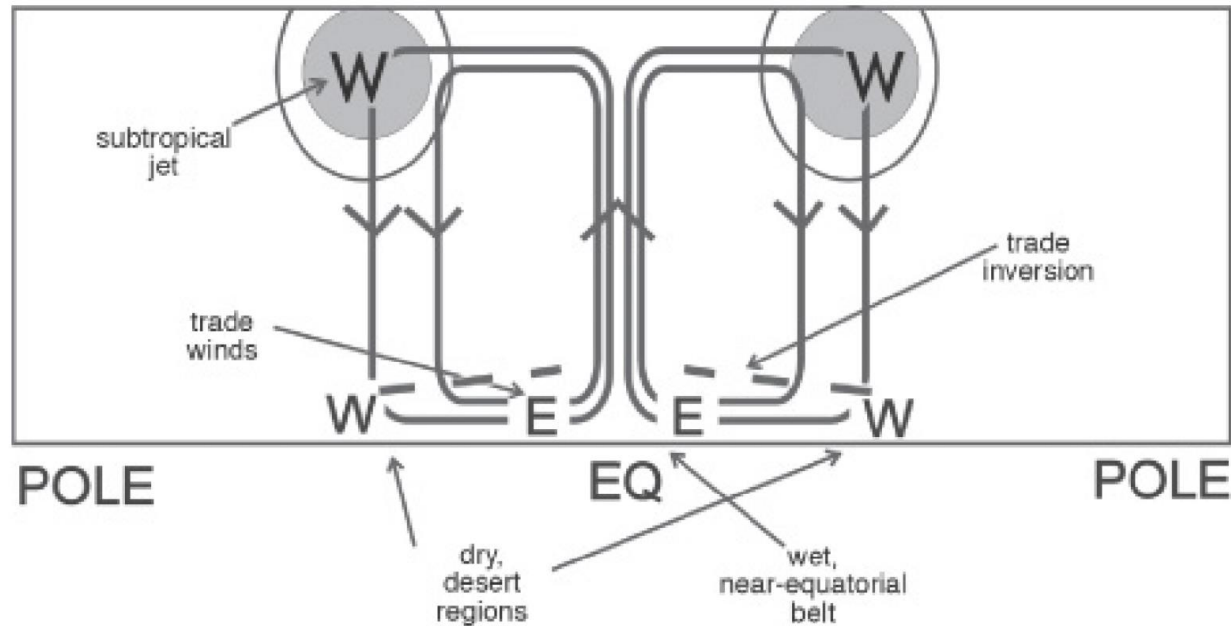


Figure 8: Schematic of the Hadley circulation (showing only the N Hem part of the circulation; there is a mirror image circulation south of the equator).

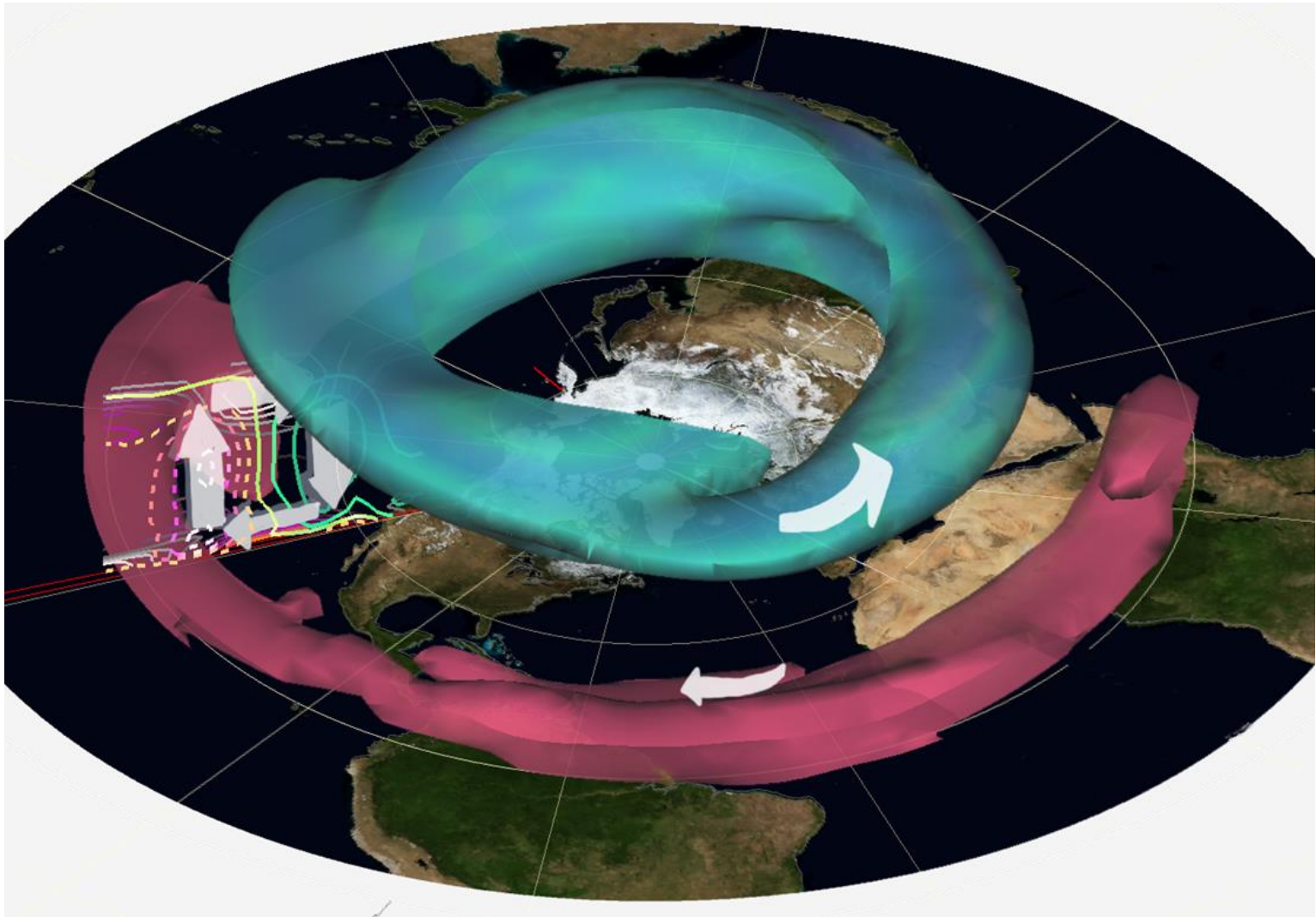
Eastward flow at upper levels, Westward flow at the surface

Hadley Cell



- ***Upward near the equator, sinks in the subtropics***
- ***Poleward & Eastward flow at upper levels, Equatorward & Westward return flow at the surface***

Hadley Cell



A 3D visualization of the Hadley Circulation

Mean meridional streamfunction

Each season, the circulation is dominated by a single cell in which air rises near the equator, flows toward the winter hemisphere at upper levels, and sinks in the subtropics

- The *Hadley cell*, named after George Hadley (1735)

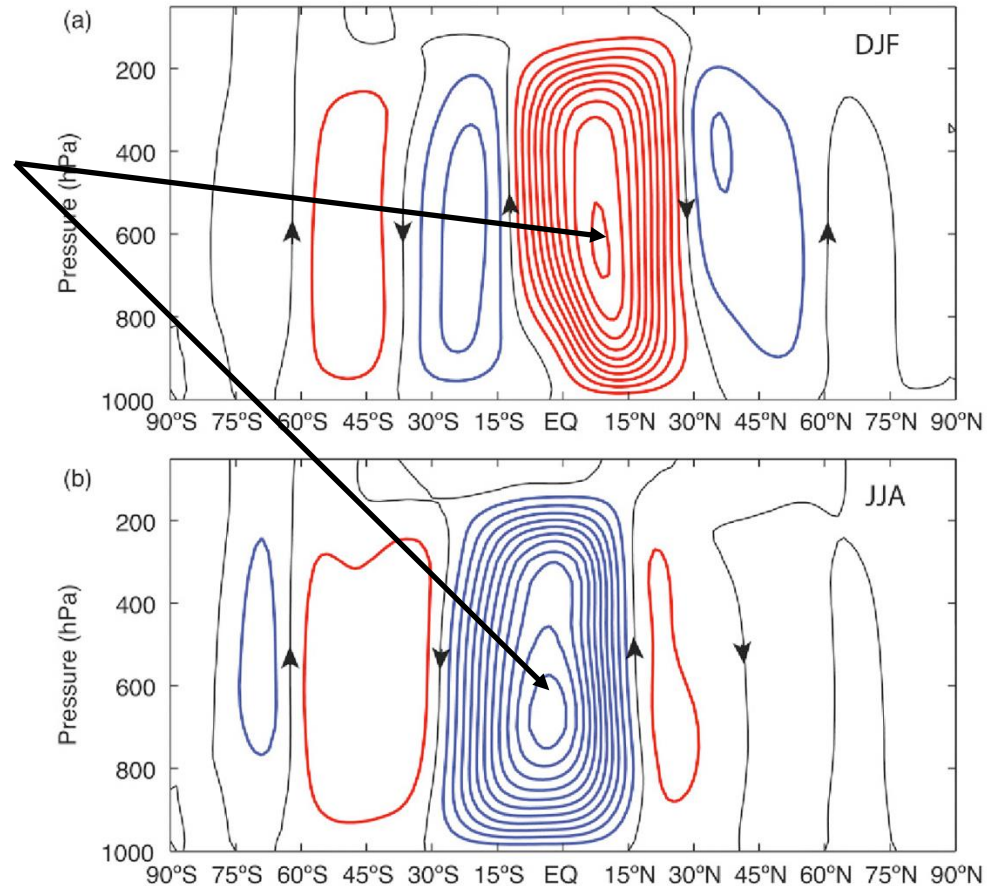


FIGURE 6.5 Latitude–pressure cross-sections of the mean meridional mass streamfunction for the (a) DJF, (b) JJA, and (c) annual mean. Contour interval is $2 \times 10^{10} \text{ kg s}^{-1}$ and the arrows on the zero contour indicate the direction of vertical motion. Red is positive and blue is negative. Based on ERA-Interim data.

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- The **Hadley cell**, named after George Hadley (1735)
- The **upward** branch of the Hadley cell occurs in the **summer hemisphere**, but cell is strongest in the **winter hemisphere**

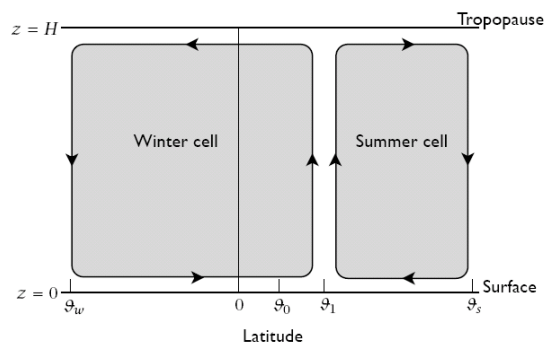


Fig. 11.9 Schematic of a Hadley circulation model when the heating is centred off the equator, at a latitude ϑ_0 . The lower level convergence occurs at a latitude ϑ_1 that is not in general equal to ϑ_0 . The resulting winter Hadley Cell is stronger and wider than the summer cell.

From Vallis (2006)

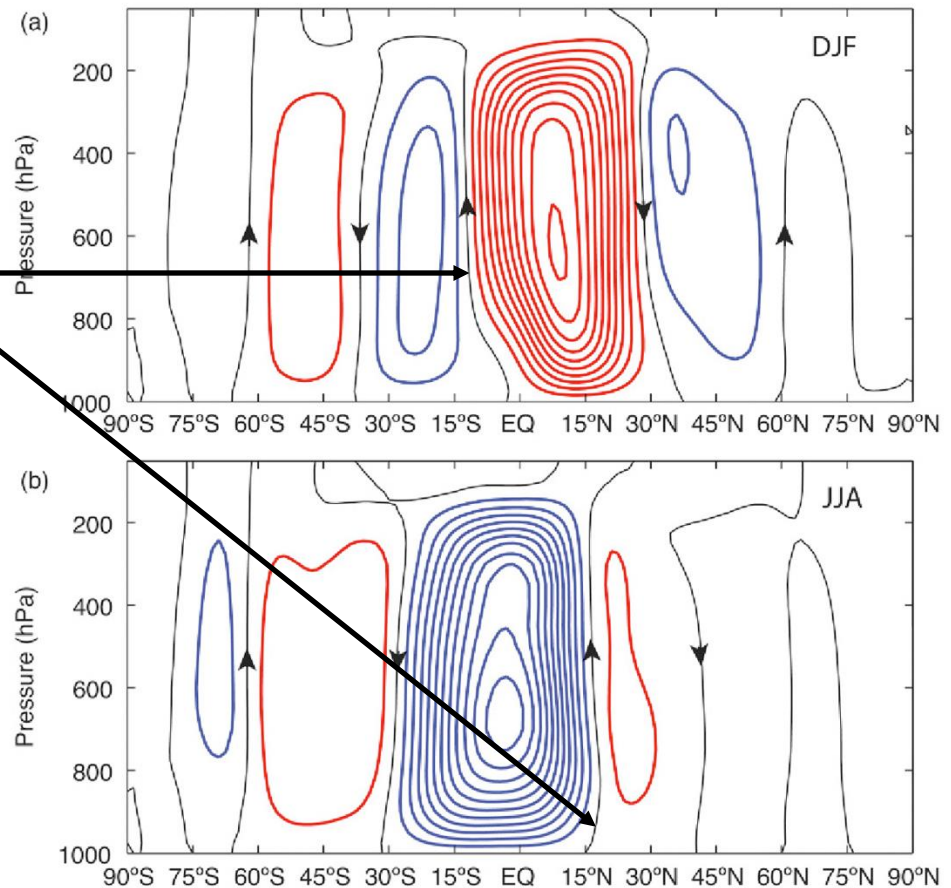


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Mean meridional streamfunction

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- The **Hadley cell**, named after George Hadley (1735)
- The **upward** branch of the Hadley cell occurs in the **summer hemisphere**, but cell is strongest in the **winter hemisphere**
- The **Ferrel cell**, named after William Ferrel (1891)
- Much weaker circulation, dominated by eddies
- Rising occurs in cold air and sinking in warmer air (indirect cell)

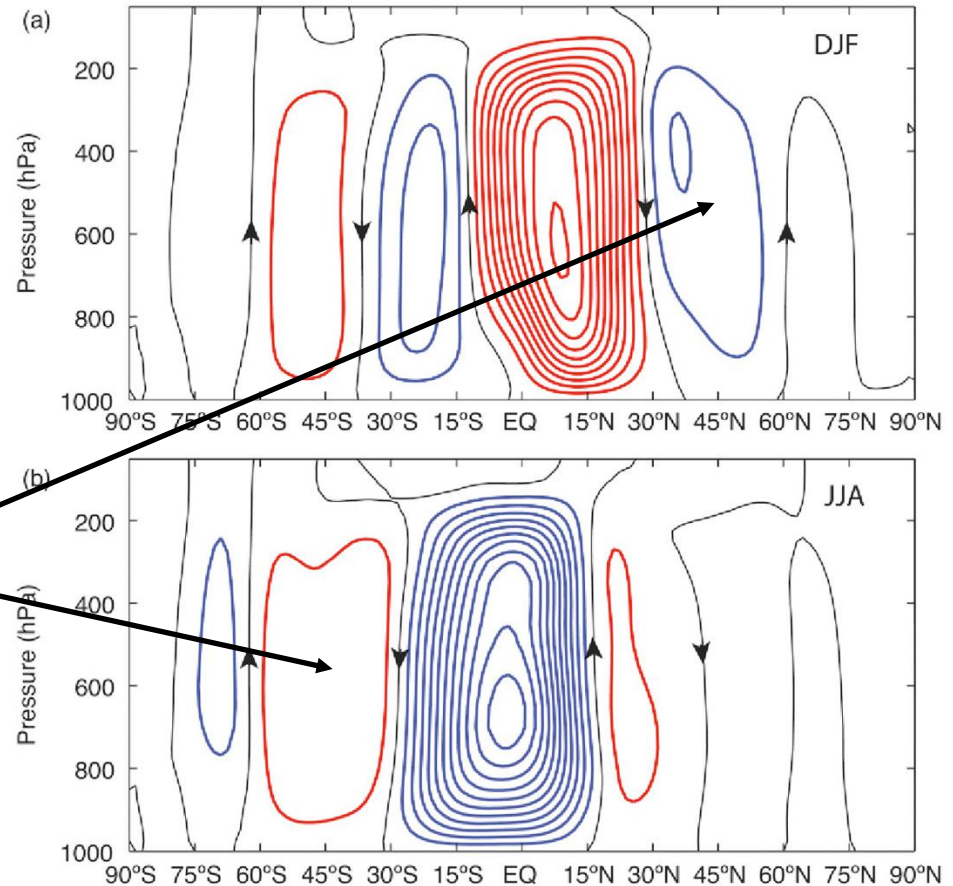


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Yearly mean meridional streamfunction

In the annual mean, more symmetric “three-cells”

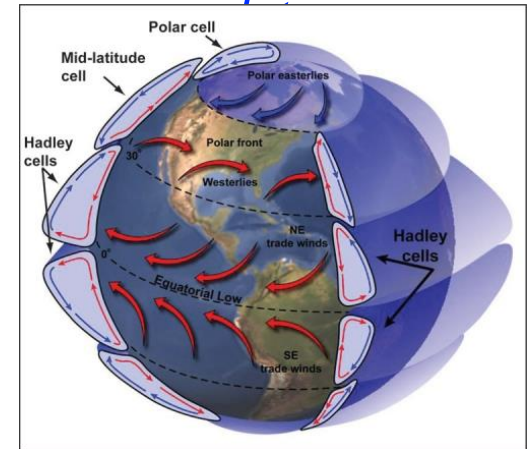
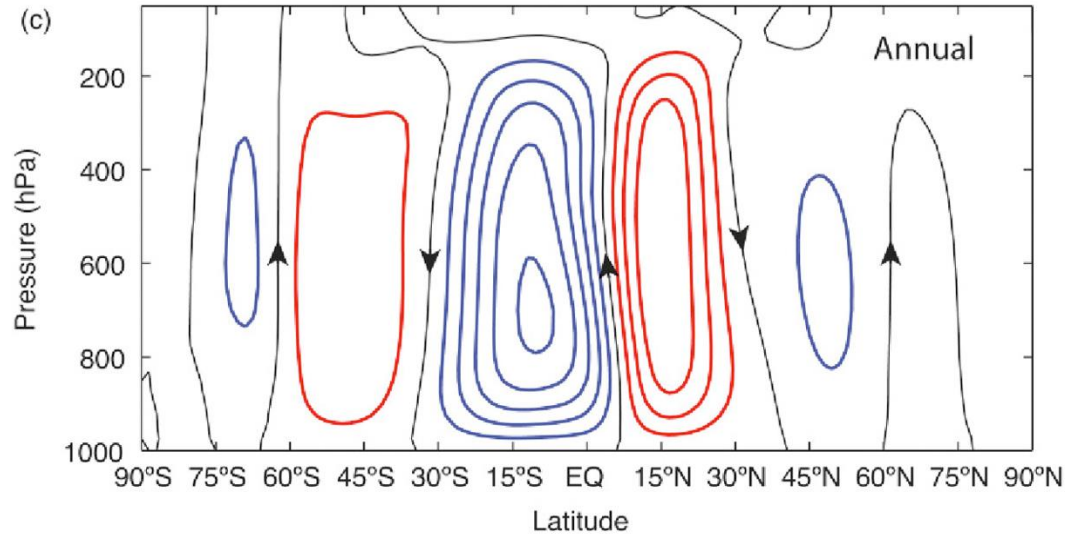


FIGURE 6.5 Latitude–pressure cross-sections of the mean meridional mass streamfunction for the (a) DJF, (b) JJA, and (c) annual mean. Contour interval is $2 \times 10^{10} \text{ kg s}^{-1}$ and the arrows on the zero contour indicate the direction of vertical motion. Red is positive and blue is negative. Based on ERA-Interim data.

In the annual mean, the rising branch is in the NH, and SH cell is stronger, implying a weak net transport of energy from the NH to the SH

Can we identify the Hadley Cell on earth?

Explore the General Circulation of the Atmosphere using climatological data on the EsGlobe <http://eddies.mit.edu/307/>

Plot zonally averaged (January and July):

- Temperature (T) and potential temperature (theta)
- Vertical velocity (omega) $\omega = Dp/Dt$
- Meridional velocity (v)
- Zonal velocity (u)

Work in groups to look for:

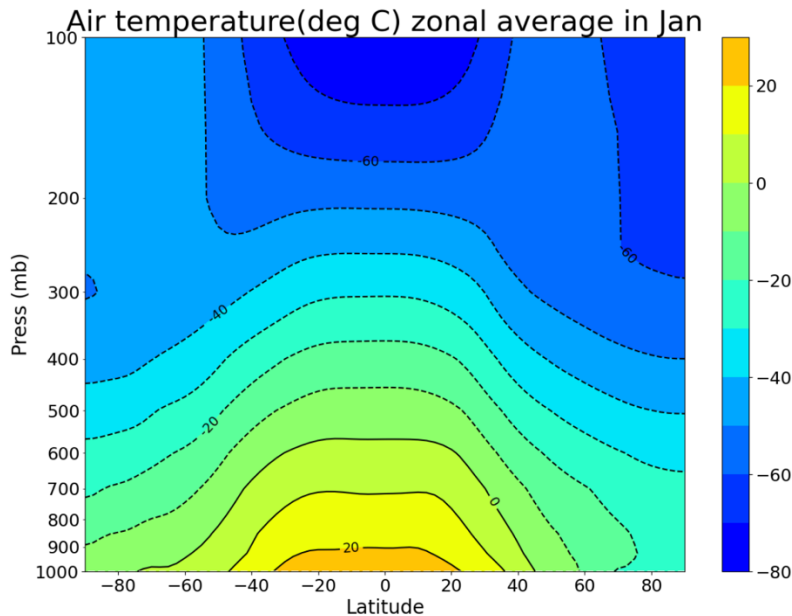
- Evidence of the Hadley cell
- Evidence of the upper-level westerlies and surface easterly (trade winds)
- Can you identify two layers?

Tip: Plot

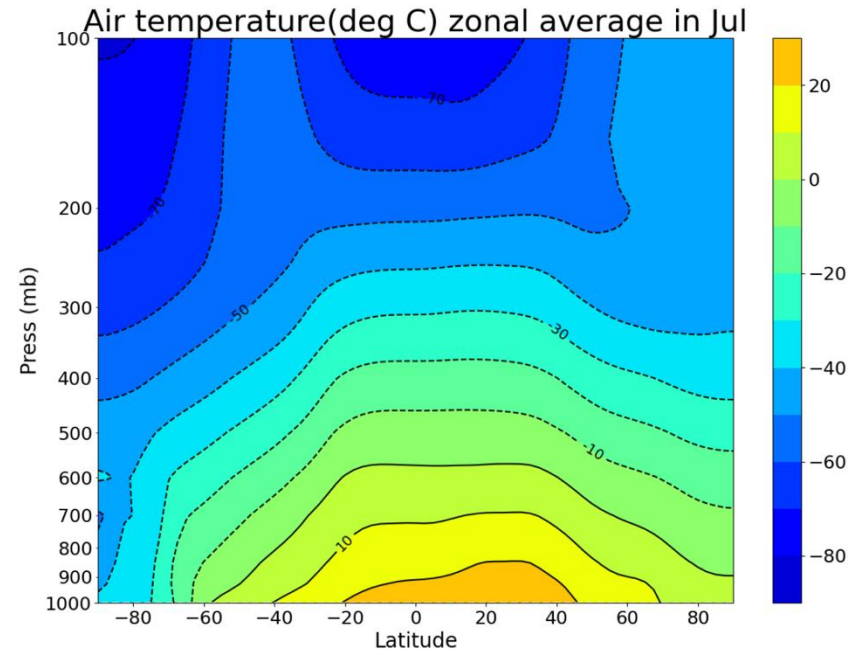
- Omega & V
- U & V

Zonal mean temperature

January



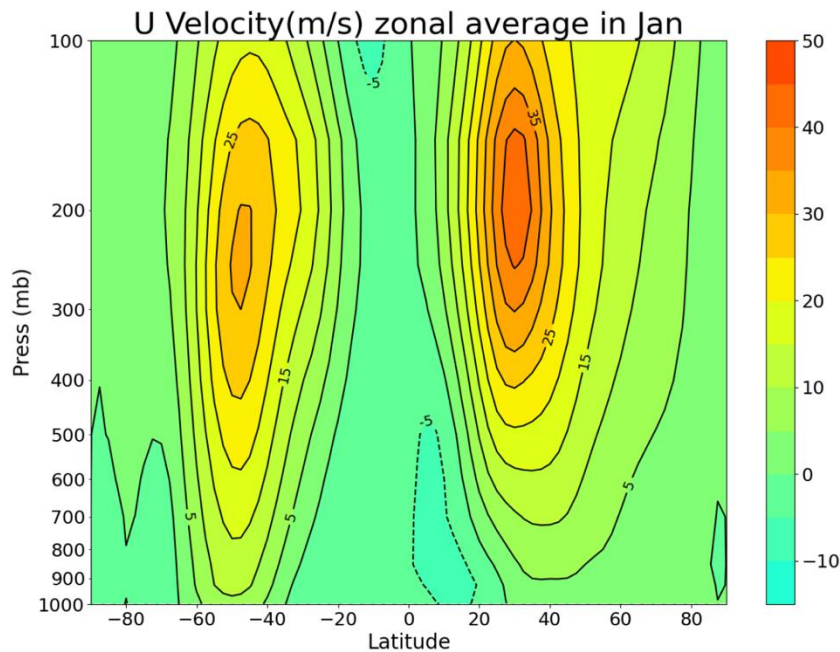
July



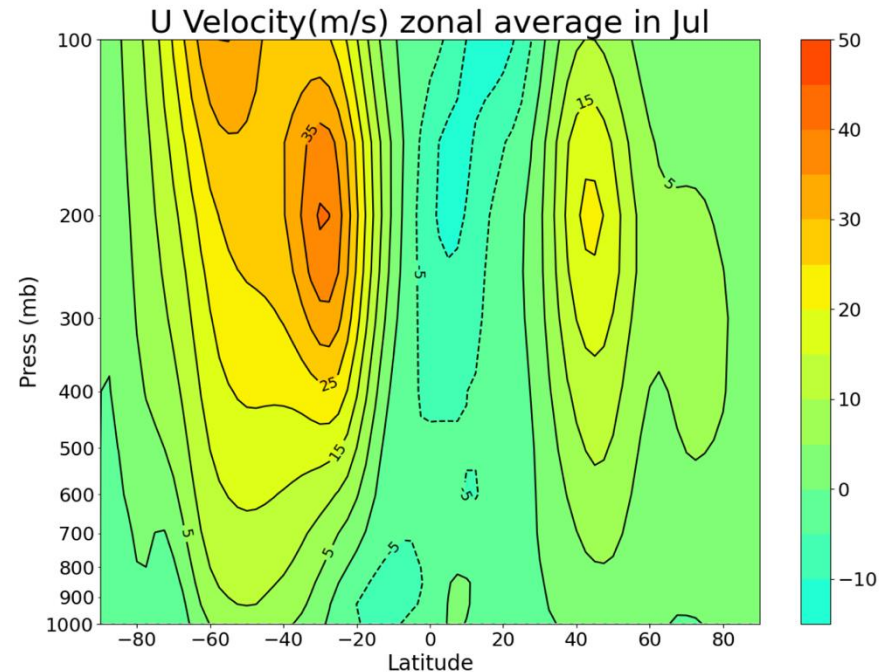
- Temperature more uniform in the tropics, decreases poleward and with height
- Where is the maximum surface temperature in each season?

Zonal mean zonal wind

January



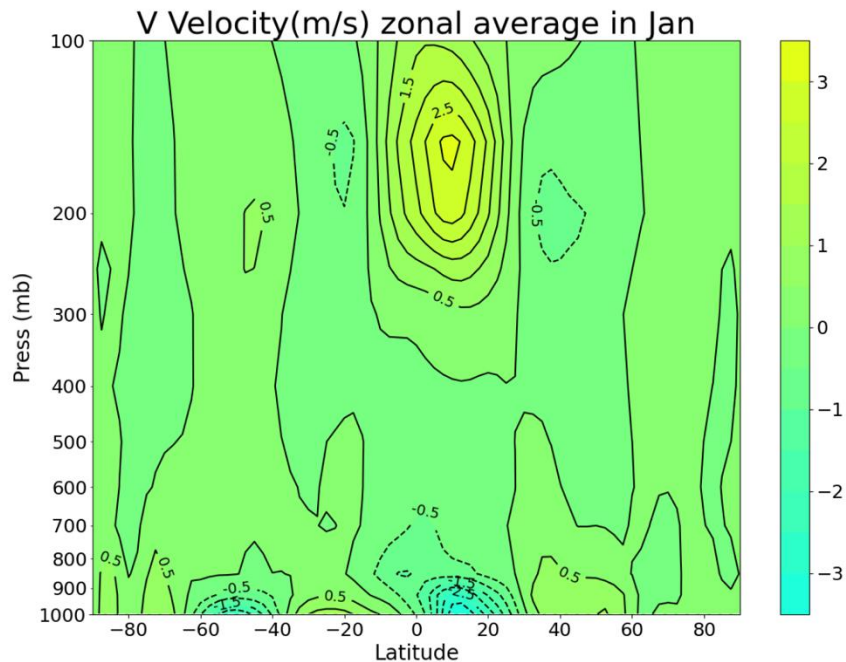
July



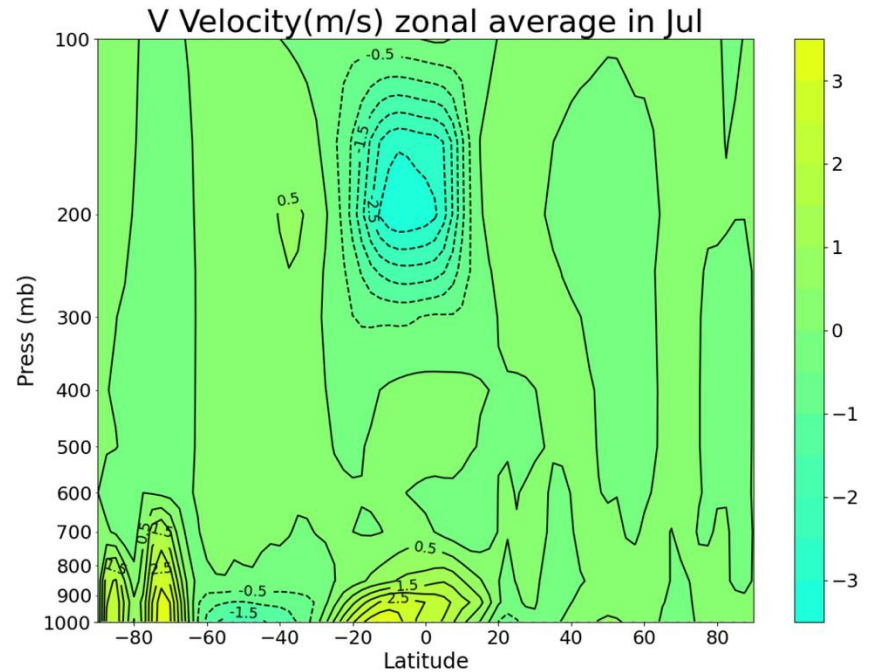
- One main jet stream in each hemisphere, eastward in both hemispheres
- Can you identify the easterlies/westerlies? Where are they in each season?

Zonal mean meridional wind

January



Jul

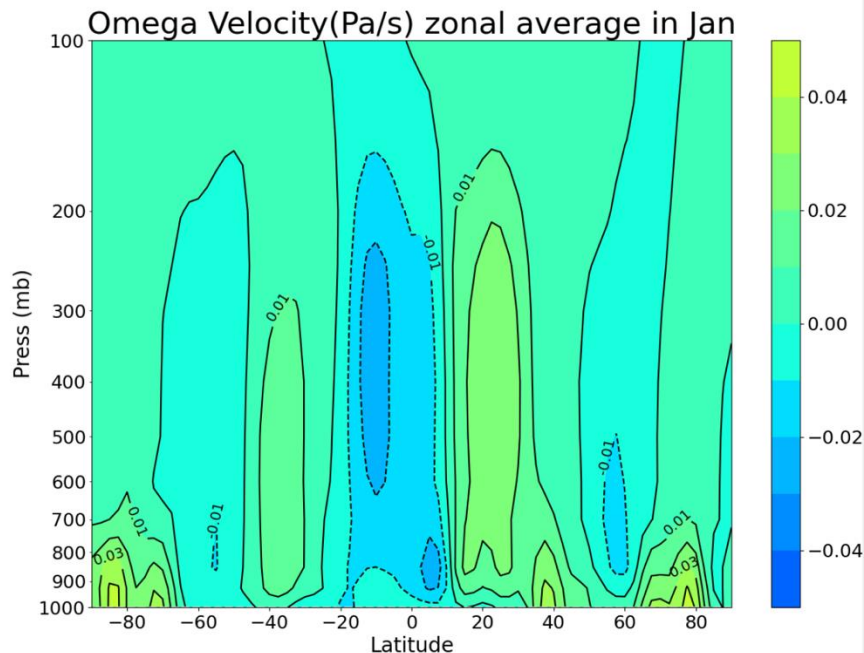


Mean V is only strong in the tropics (Hadley Cell), and mostly confined to either upper level or to the surface-

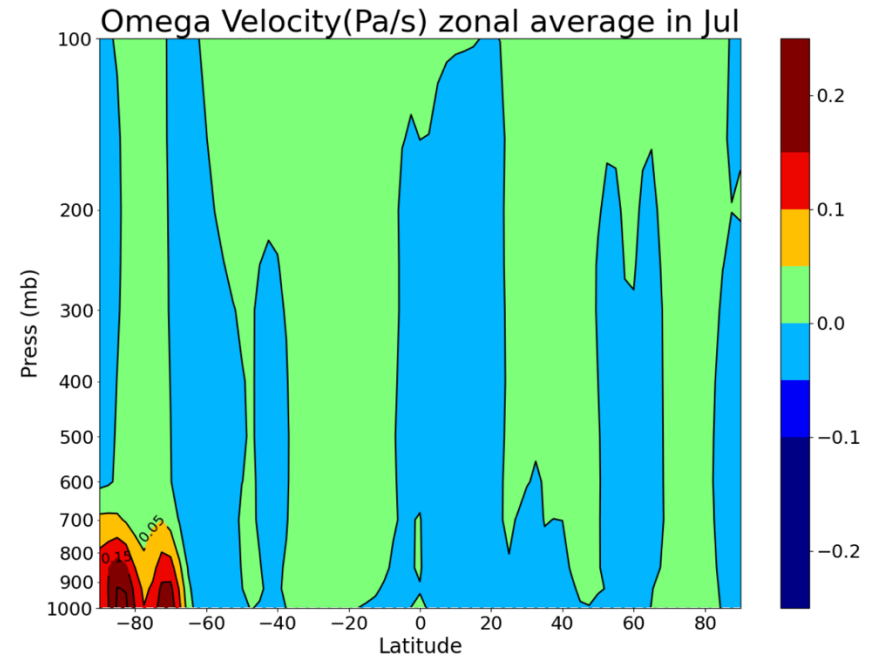
~Two layers!

Zonal mean vertical wind

January



July

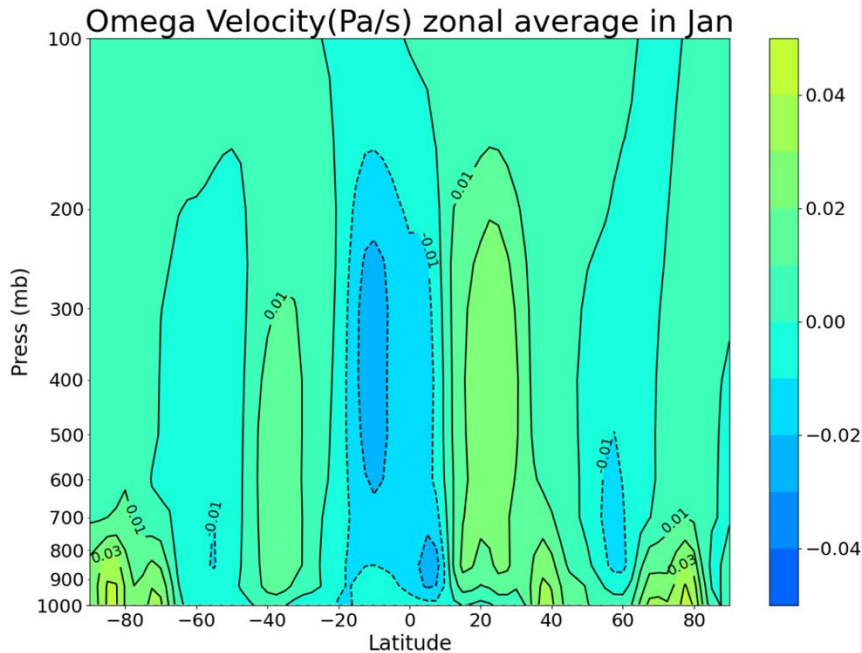


Note: $w = \frac{Dz}{Dt} \left[\frac{m}{sec} \right]$ and $\omega = \frac{DP}{Dt} \left[\frac{Pa}{sec} \right] \rightarrow \omega = -\rho g w$

- Where is the air ascending/descending in each season?
- Where do you expect to find deserts?

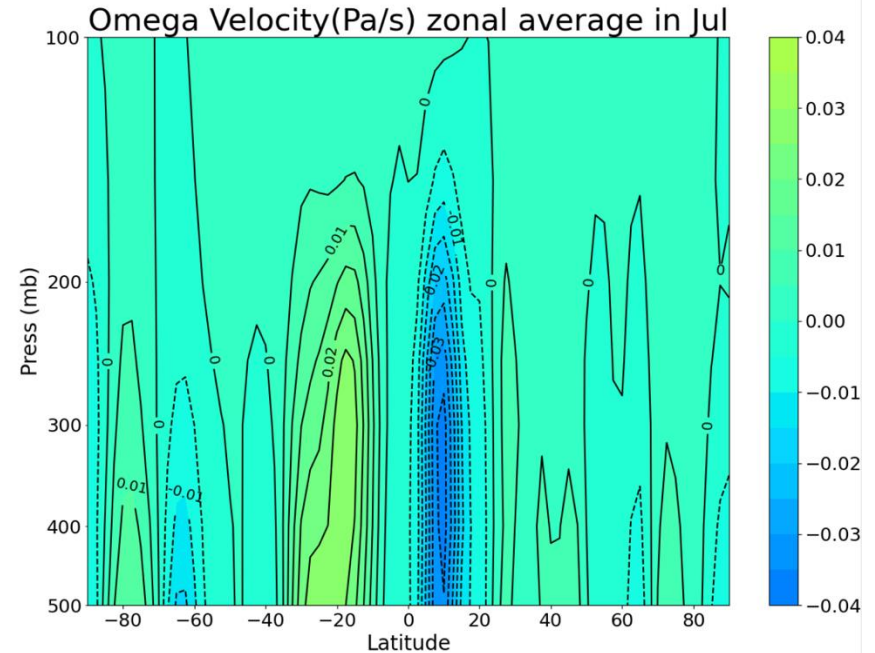
Zonal mean vertical wind

January



July

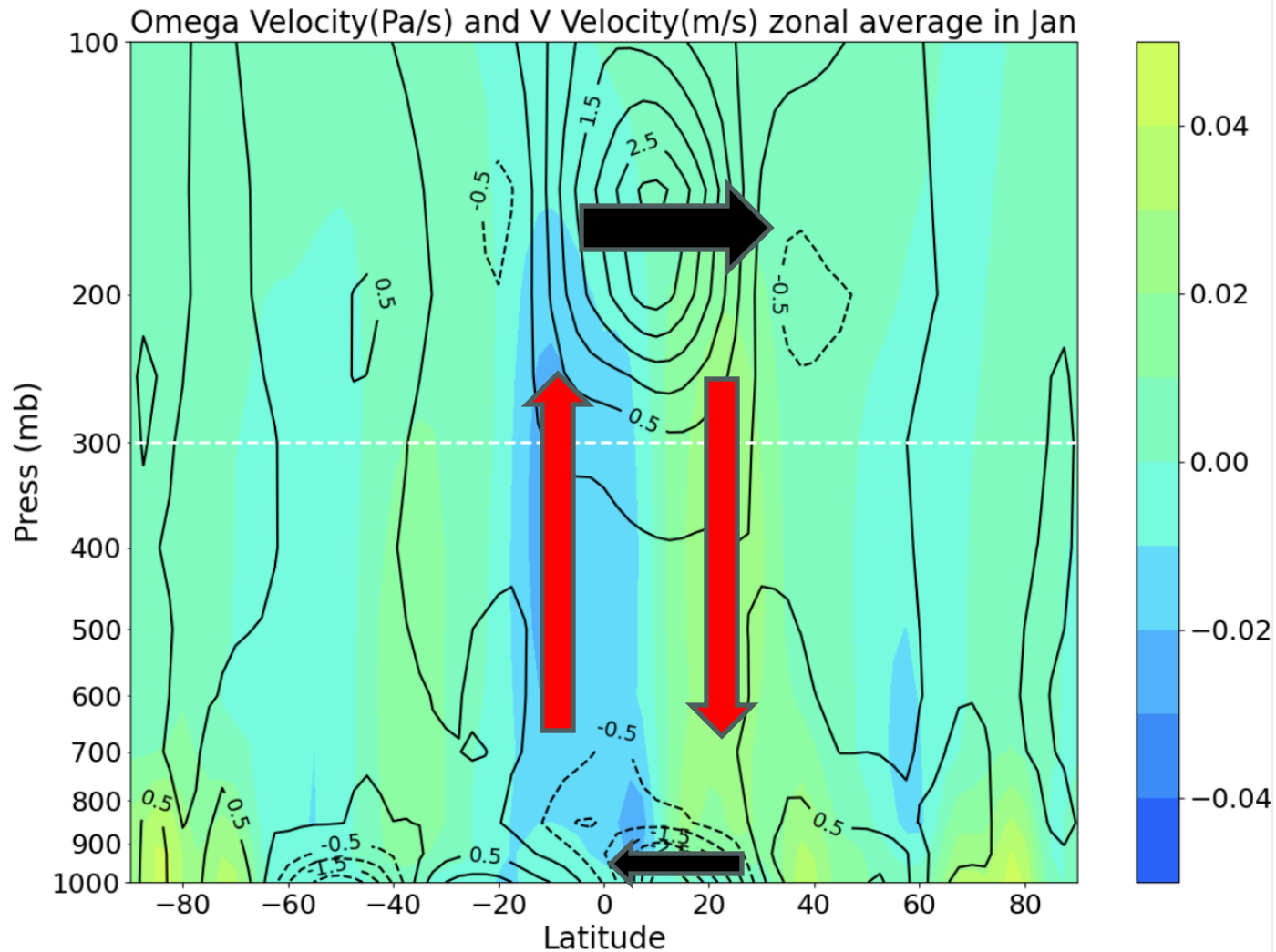
Zoom in
500-100 mb



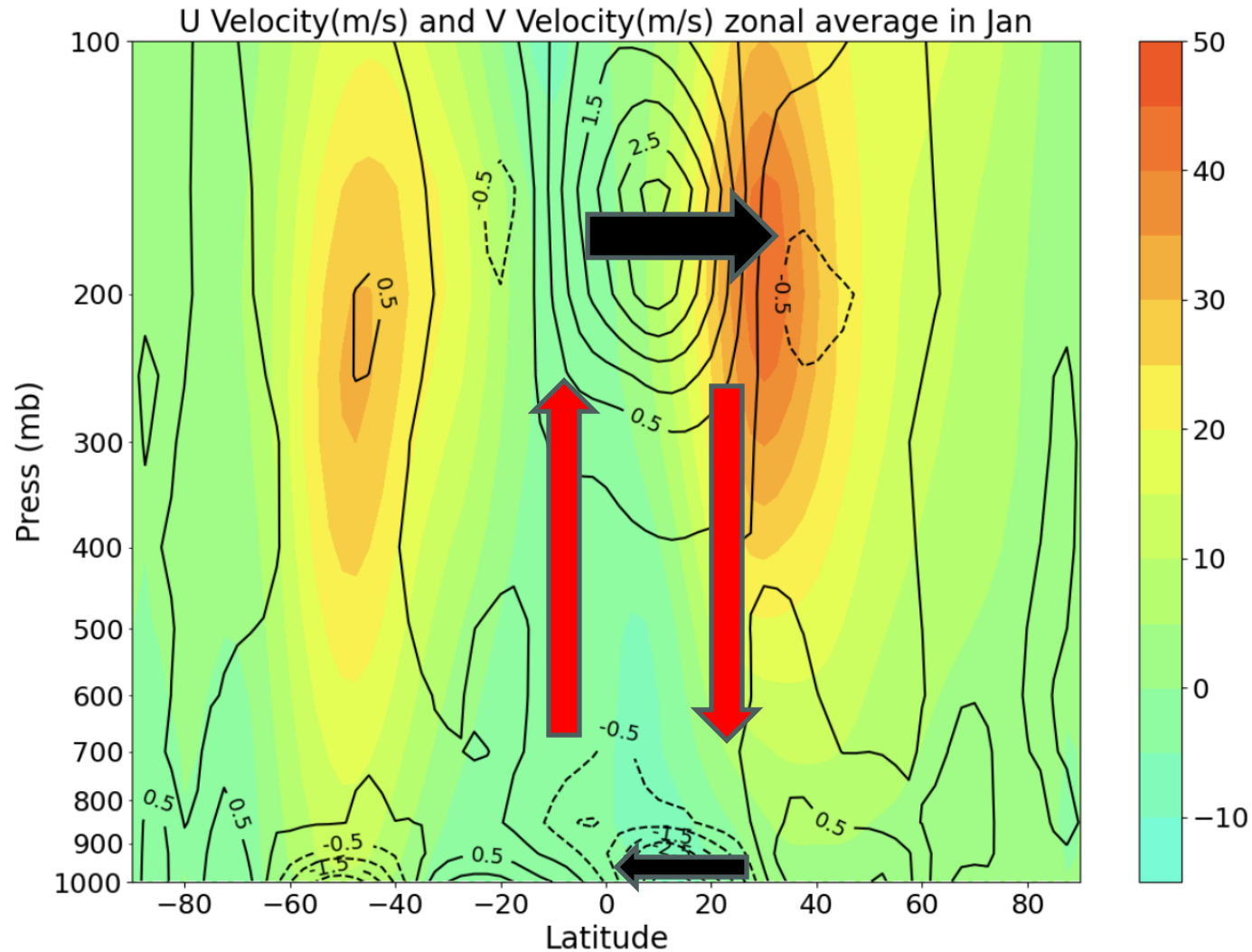
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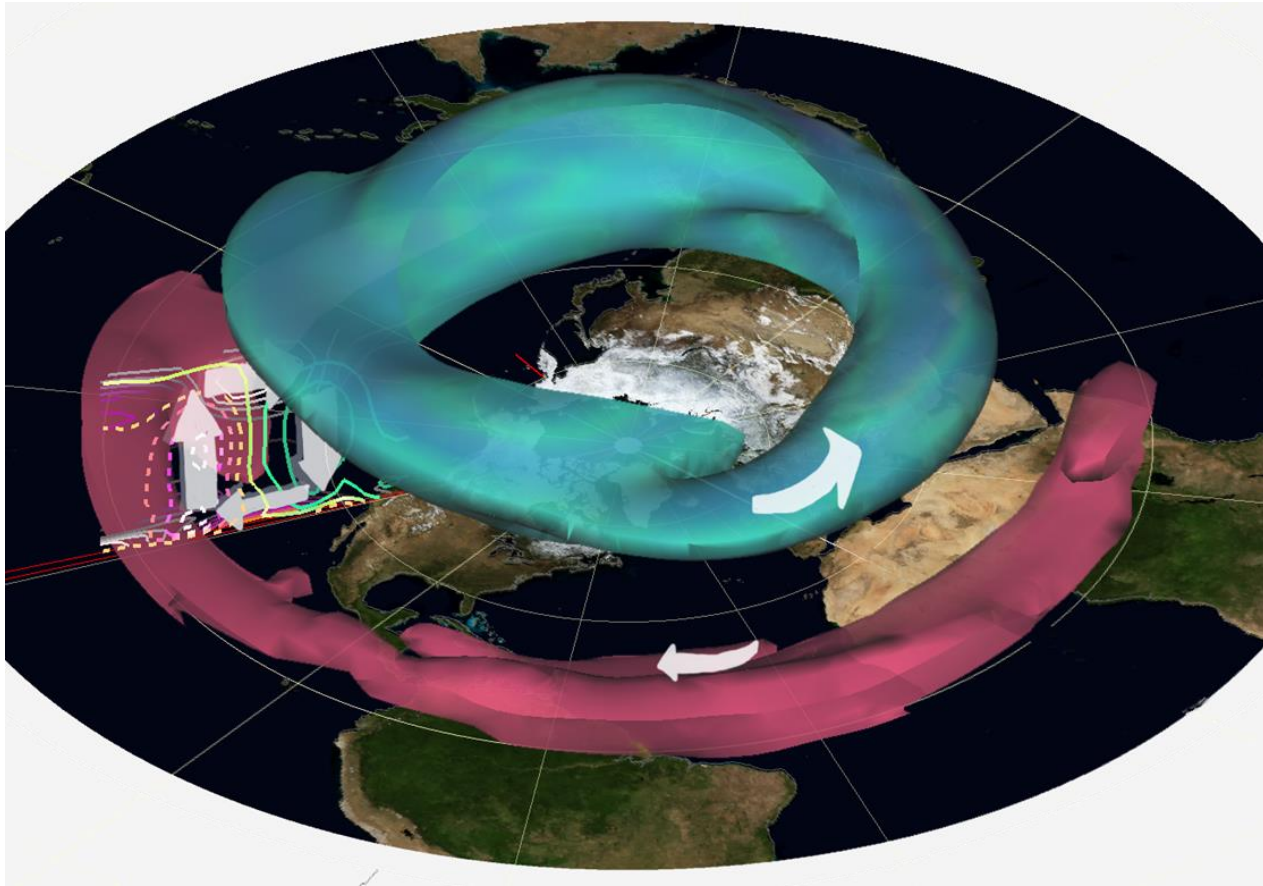
- Where is the air ascending/descending in each season?
- Where do you expect to find deserts?

Hadley Circulation! V and W superimposed



Hadley Circulation! V and U superimposed





Can we see evidence for this circulation in the atmosphere?

Yes!

Let's estimate the poleward heat flux that this circulation transports

$$\begin{aligned}\overline{\mathcal{H}}_{atmos}^{\lambda} &= \iint \rho v E \, dA \\ &= a \cos \varphi \int_0^{2\pi} \int_0^{\infty} \rho v E \, dz \, d\lambda \\ \boxed{\rho dz = -dp/g} &\longrightarrow = \frac{a}{g} \cos \varphi \int_0^{2\pi} \int_0^{p_s} v E \, dp \, d\lambda ,\end{aligned}$$

Note:

- Here we used hydrostatic balance to replace dz with dp
- The integral is from $p=0$ (top of the atmosphere) to the surface: $p_s = 1000mb = 10^5 \text{ Pa}$
- The $2\pi a \cos \varphi$ is related to the zonal averaging at latitude φ , i.e. $2\pi r$ at radius $r = a \cos \varphi$

Before that, let's introduce an important quantity-

Potential temperature:

The temperature that a parcel will acquire if it were compressed adiabatically from p and T to a standard pressure p_0

$$\theta = T \left(\frac{p_0}{p} \right)^\kappa$$

with $k = R/c_p = 2/7$ and conventionally $p_0 = 1000mb$.

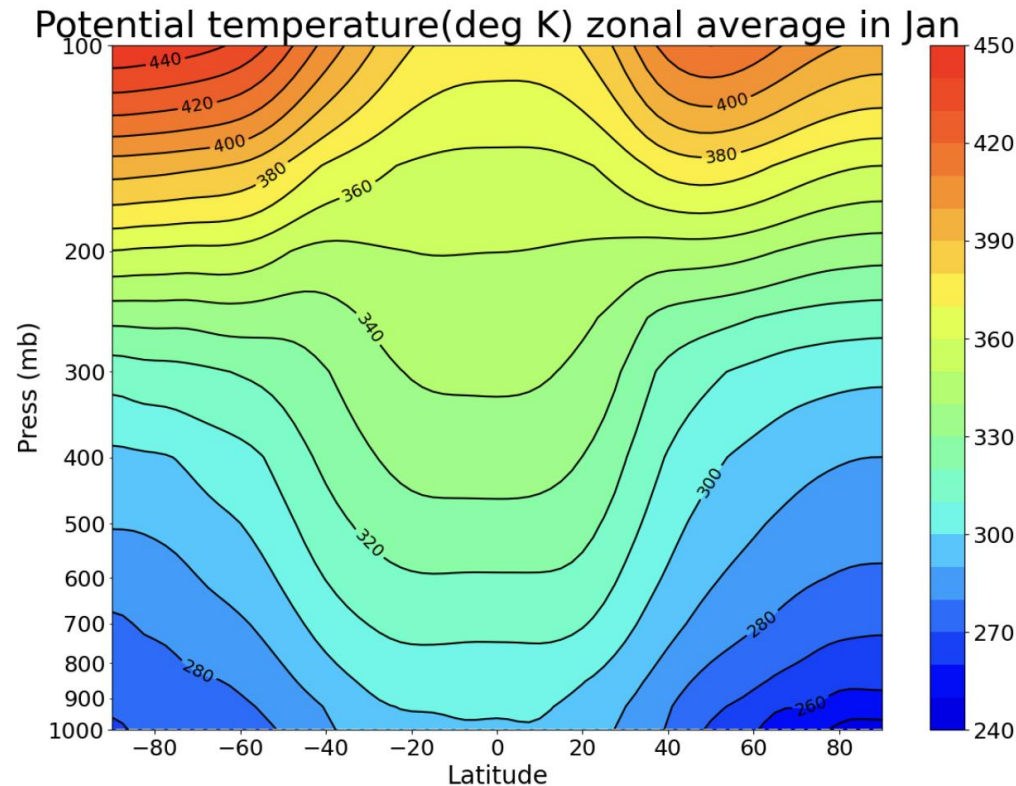
In adiabatic conditions, it follows that:

$$\frac{d\theta}{\theta} = \frac{dT}{T} - k \frac{dp}{p} = 0$$

Potential temperature is conserved in an adiabatic process!

Note that for an adiabatic process, it is also exact to write $C_p\theta = C_pT + gz$

Potential temperature: $\theta = T \left(\frac{p_0}{p} \right)^\kappa$



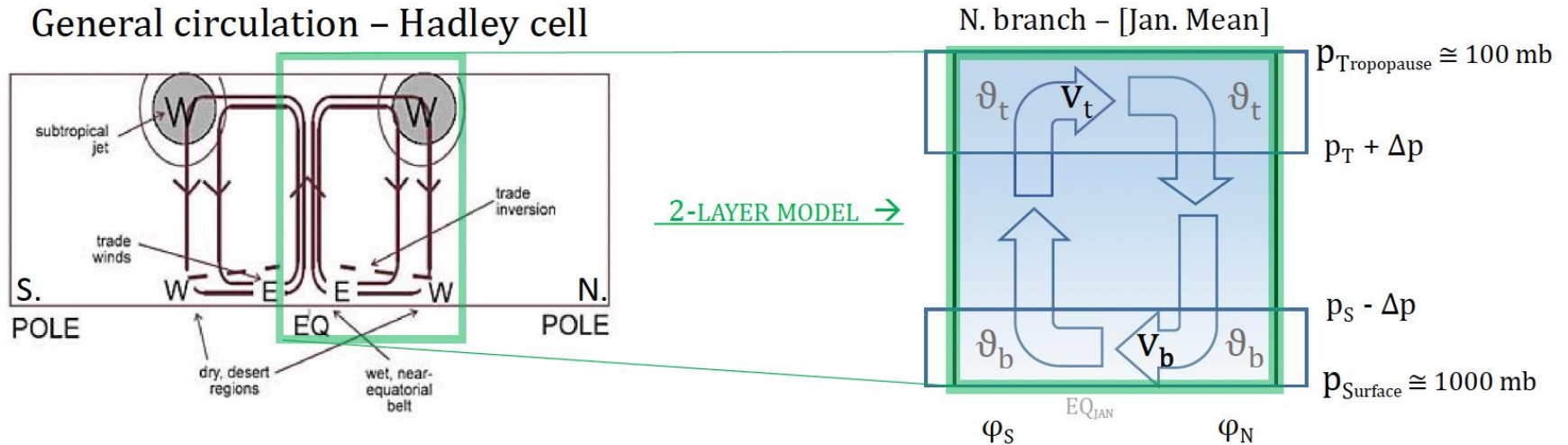
- Theta increases with height → Stably stratified
- Still increases in the stratosphere, but much stronger gradient
- Horizontal gradients are small in the tropics

Meridional Heat transport

$$\begin{aligned}\mathcal{H} &= \rho c_p \int_0^{\infty} \oint \bar{v} \bar{\vartheta} dx dz = \frac{c_p}{g} \int_0^{p_s} \oint \bar{v} \bar{\vartheta} dx dp \\ &= \frac{c_p}{g} \times 2\pi a \cos \varphi \int_0^{p_s} [\bar{v} \bar{\vartheta}] dp\end{aligned}$$

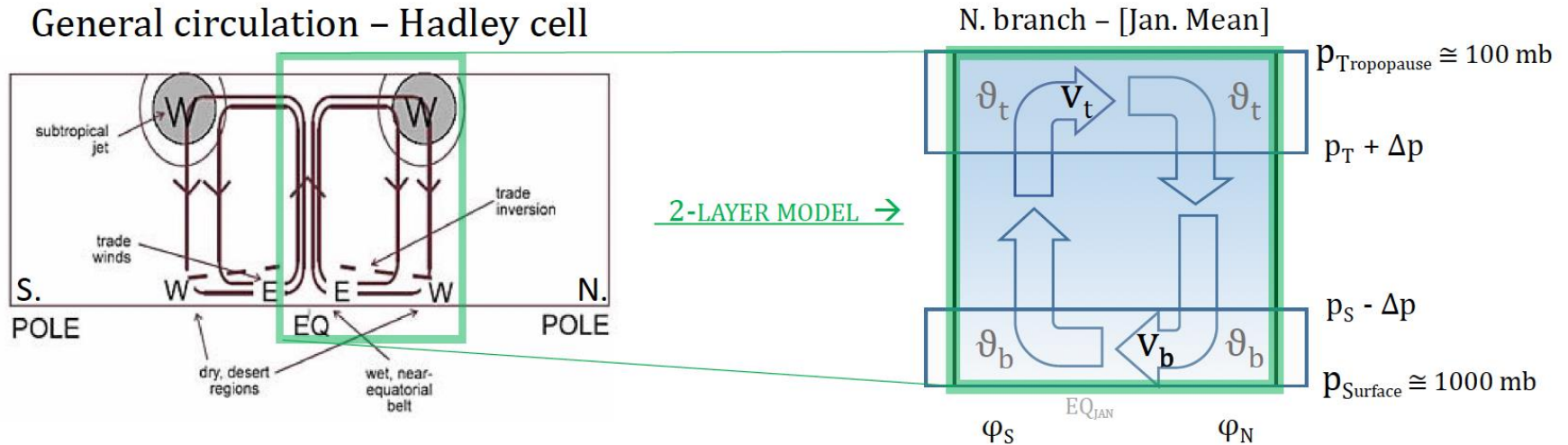
Meridional transport=The mean meridional velocity times heat (potential temperature) integrated vertically

The two-layer model for Heat transport in the tropics



The only contributions are at upper levels (poleward flow) and at low-levels (equatorward flow)

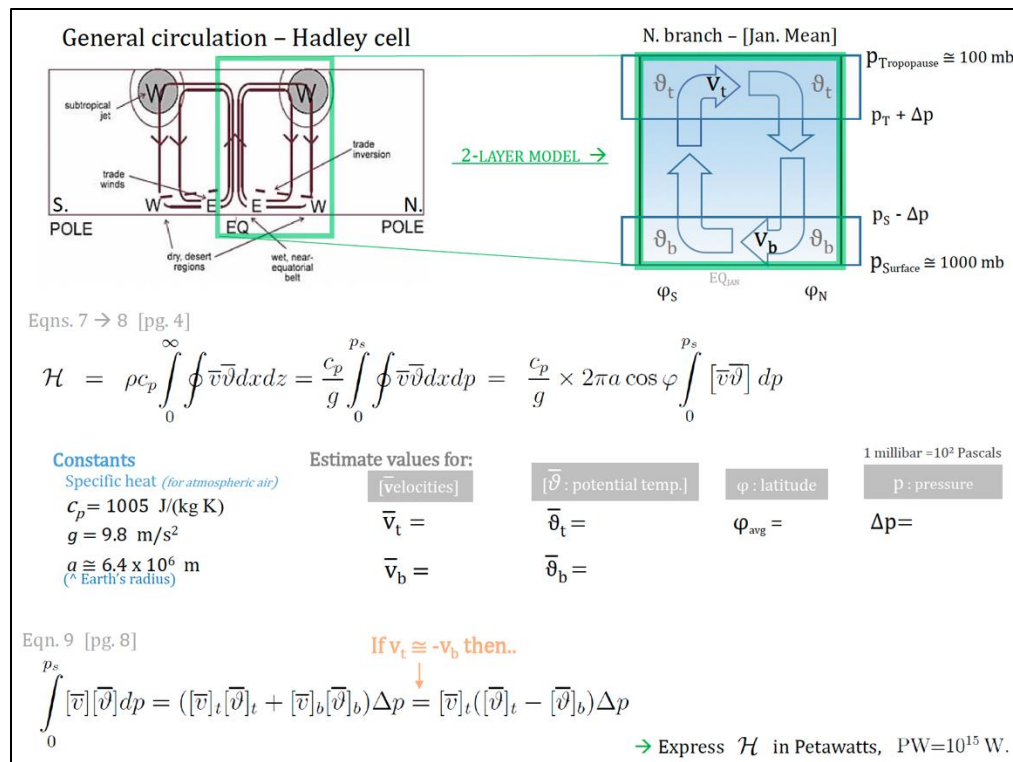
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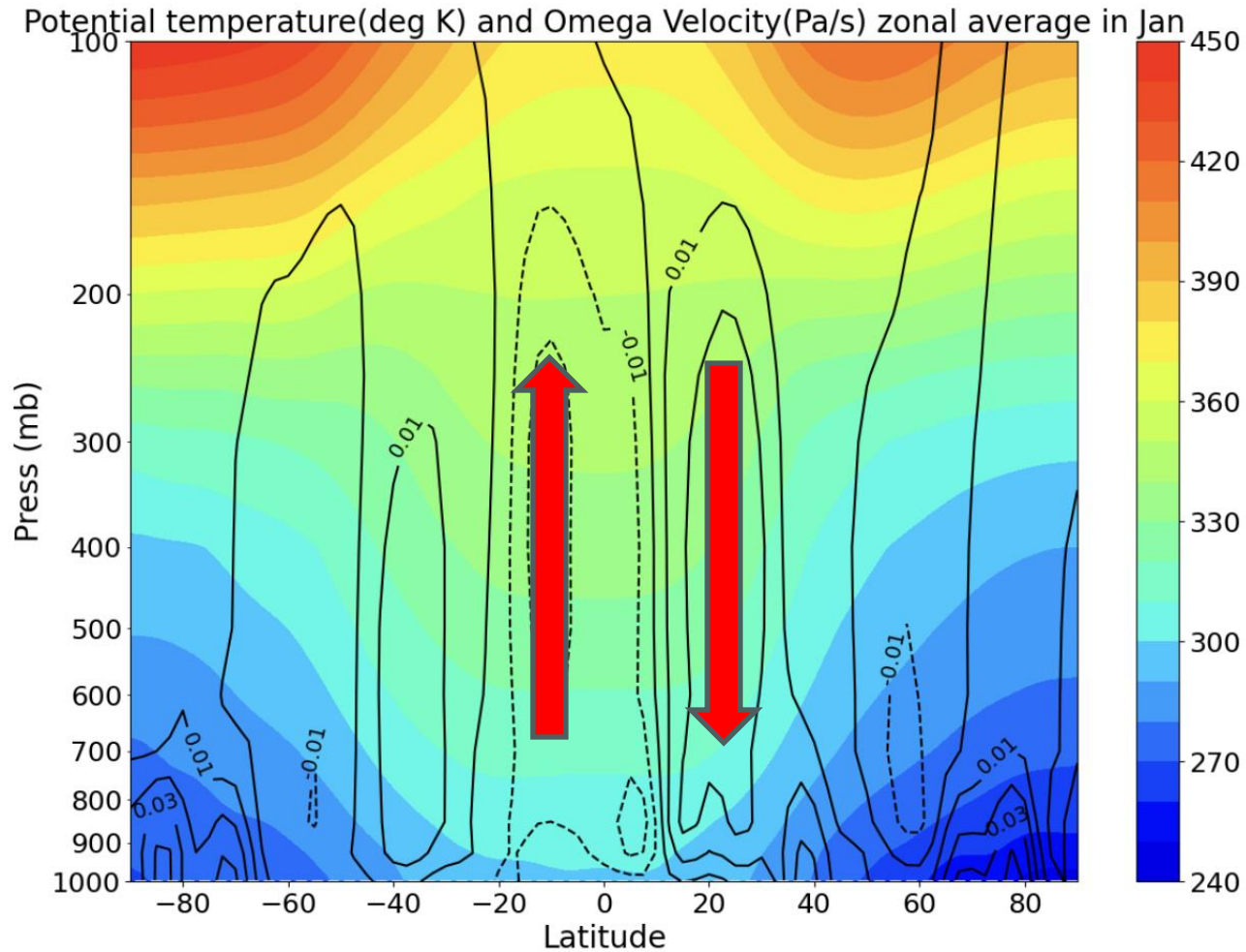
The only contributions are at upper levels (poleward flow) and at low-levels (equatorward flow)

$$\int_0^{p_s} [\bar{v}][\bar{\vartheta}] dp = ([\bar{v}]_t[\bar{\vartheta}]_t + [\bar{v}]_b[\bar{\vartheta}]_b) \Delta p = [\bar{v}]_t([\bar{\vartheta}]_t - [\bar{\vartheta}]_b) \Delta p$$

Exercise: estimate the poleward heat flux using the EsGlobe and the schematic

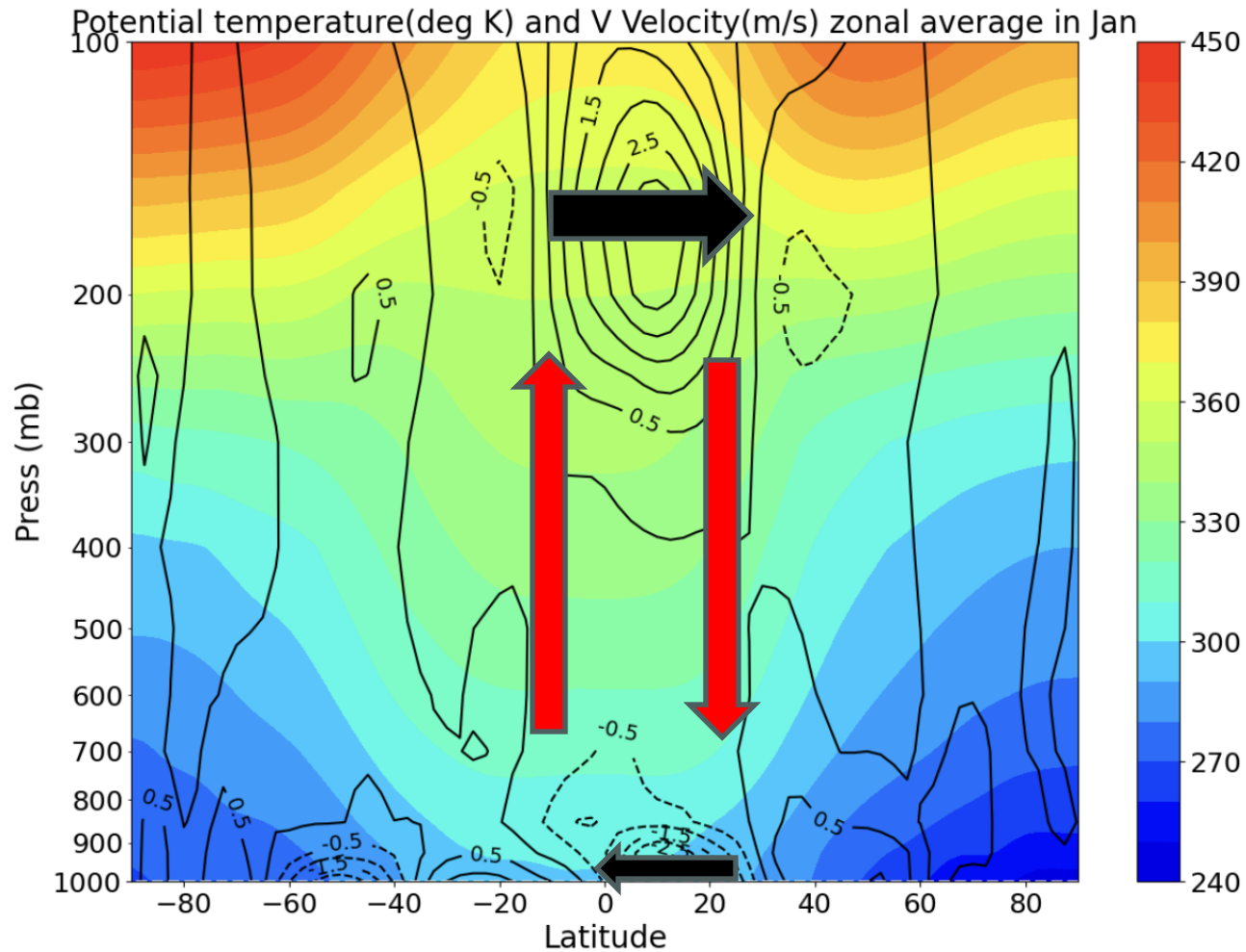


Two layer model to estimate the heat transport



Tip: plot potential temperature + vertical velocity

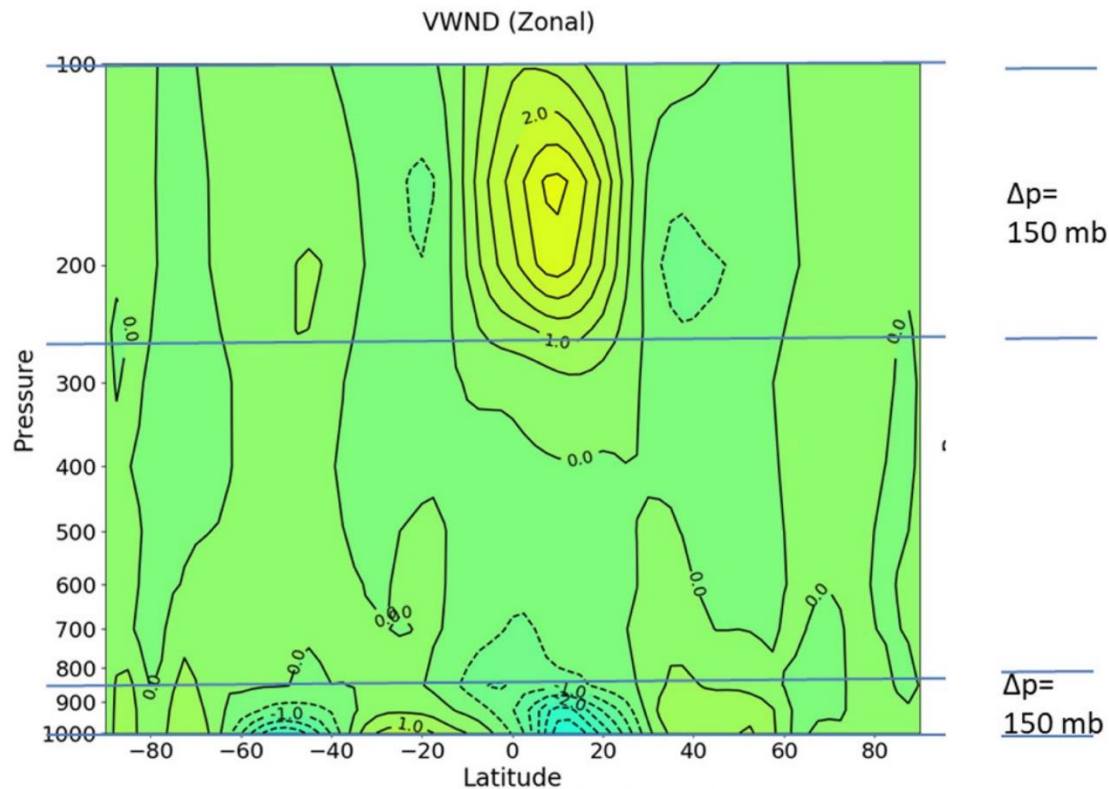
Two layer model to estimate the heat transport



Tip: plot potential temperature + meridional velocity

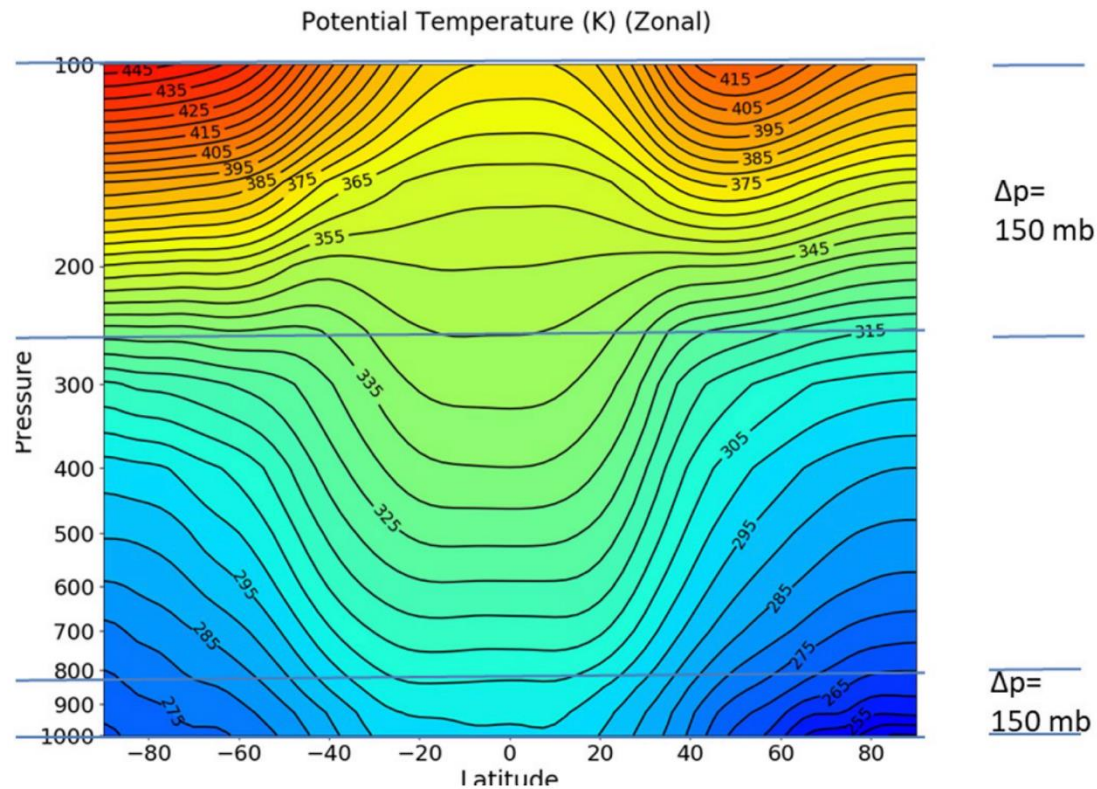
Two-layer model for heat transport in the Hadley Cell

$$\int_0^{p_s} [\bar{v}][\bar{\vartheta}] dp = ([\bar{v}]_t[\bar{\vartheta}]_t + [\bar{v}]_b[\bar{\vartheta}]_b)\Delta p = [\bar{v}]_t([\bar{\vartheta}]_t - [\bar{\vartheta}]_b)\Delta p.$$



Two-layer model for heat transport in the Hadley Cell

$$\int_0^{p_s} [\bar{v}][\bar{\vartheta}] dp = ([\bar{v}]_t[\bar{\vartheta}]_t + [\bar{v}]_b[\bar{\vartheta}]_b)\Delta p = [\bar{v}]_t([\bar{\vartheta}]_t - [\bar{\vartheta}]_b)\Delta p.$$



Meridional Heat transport- two-layer approach:

$$\begin{aligned}\mathcal{H} &= \rho c_p \int_0^\infty \oint \bar{v} \bar{\vartheta} dx dz = \frac{c_p}{g} \int_0^{p_s} \oint \bar{v} \bar{\vartheta} dx dp \\ &= \frac{c_p}{g} \times 2\pi a \cos \varphi \int_0^{p_s} [\bar{v} \bar{\vartheta}] dp\end{aligned}$$

$$\int_0^{p_s} [\bar{v}] [\bar{\vartheta}] dp = ([\bar{v}]_t [\bar{\vartheta}]_t + [\bar{v}]_b [\bar{\vartheta}]_b) \Delta p = [\bar{v}]_t ([\bar{\vartheta}]_t - [\bar{\vartheta}]_b) \Delta p.$$

$$\begin{aligned}H &= [(10^3)/10] \times (6 \times 6 \times 10^6) \times (3 \times 50) \times (1.5 \times 10^4) \\ &= 8 \times 10^{15} \text{ W} = 8 \text{ PW}\end{aligned}$$

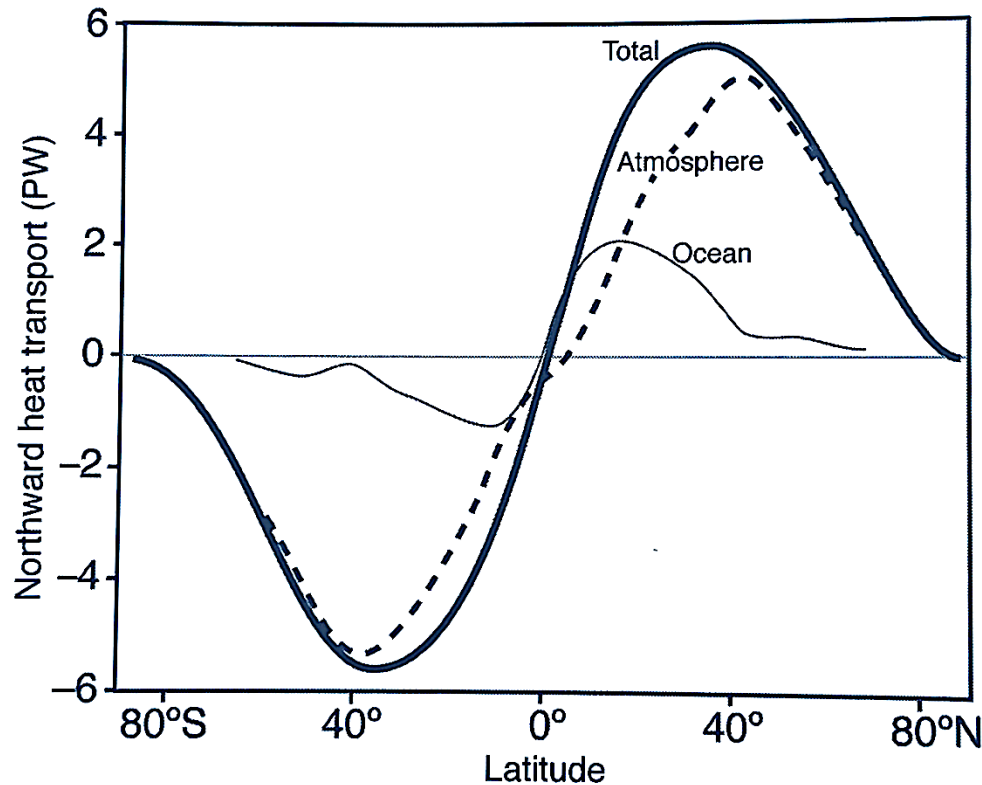
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$$\begin{aligned}\mathcal{H} &= \rho c_p \int_0^\infty \oint \bar{v} \bar{\vartheta} dx dz = \frac{c_p}{g} \int_0^{p_s} \oint \bar{v} \bar{\vartheta} dx dp \\ &= \frac{c_p}{g} \times 2\pi a \cos \varphi \int_0^{p_s} [\bar{v} \bar{\vartheta}] dp\end{aligned}$$

$$\int_0^{p_s} [\bar{v}] [\bar{\vartheta}] dp = ([\bar{v}]_t [\bar{\vartheta}]_t + [\bar{v}]_b [\bar{\vartheta}]_b) \Delta p = [\bar{v}]_t ([\bar{\vartheta}]_t - [\bar{\vartheta}]_b) \Delta p.$$

$$H \approx 8 \text{ PW!}$$

Compare our estimates to reanalysis estimates:

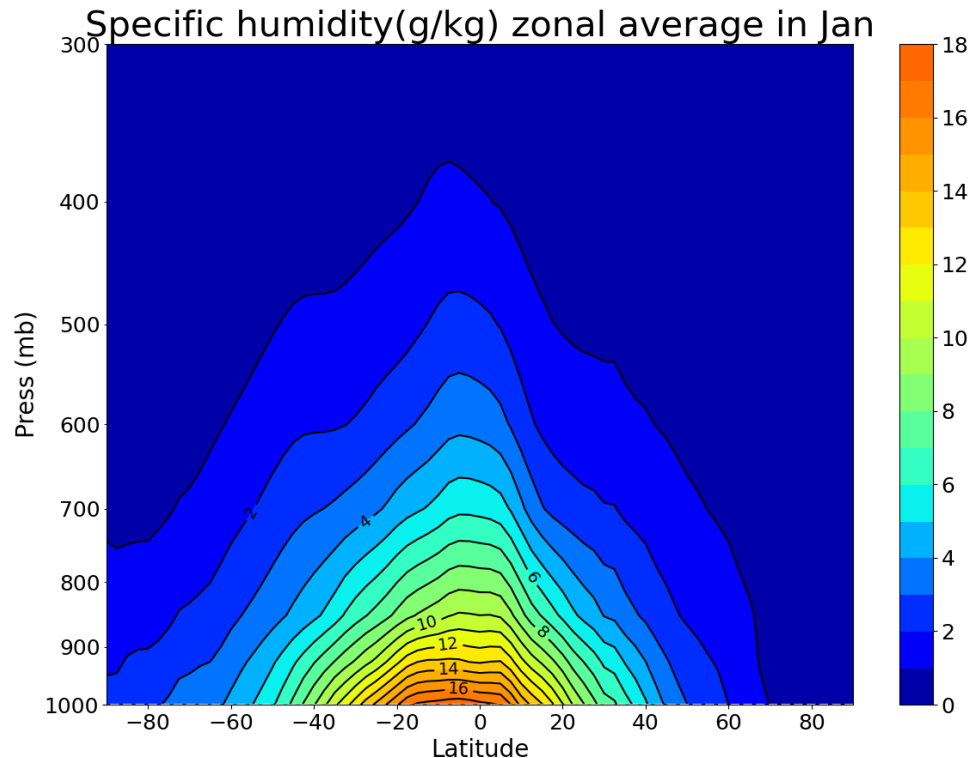


Why do we get 8 PW?? seems too high!

We need to consider also latent heating...

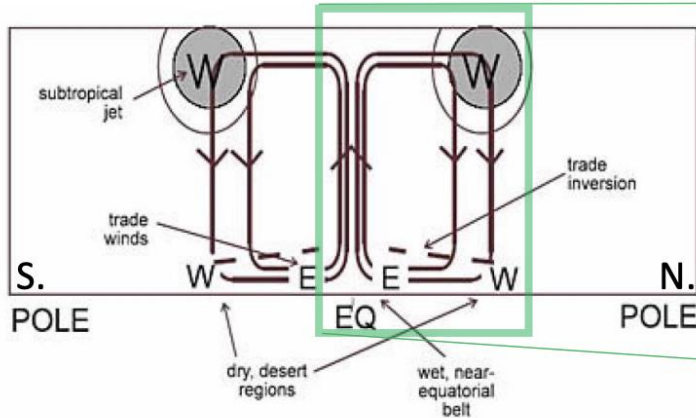
Moisture distribution

Plot zonally averaged field of **specific humidity (g/kg)**

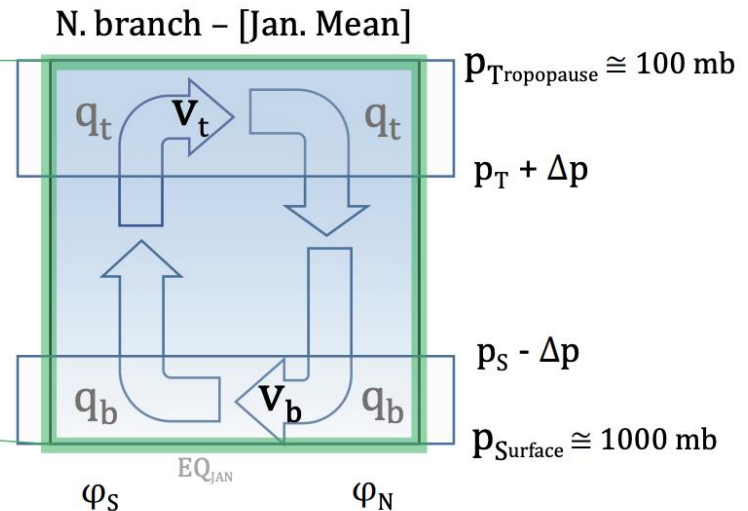


- **Specific humidity**= the amount of water vapour (in grams) in 1kg of air
- Confined to low-levels, higher in tropics

General circulation – Hadley cell



2-LAYER MODEL →



Latent Heat Flux Contribution Eqn. 10 [pg. 9] If $q_t \approx 0$ then...

$$\mathcal{L} = \frac{L}{g} \times 2\pi a \cos \varphi \int_0^{p_s} \bar{v} \bar{q} dp \simeq \frac{L}{g} \times 2\pi a \cos \varphi [\bar{v}]_b [\bar{q}]_b \Delta p.$$

Constants (for atmospheric air)

Latent heat of fusion

$$L = 2.25 \times 10^6 \text{ J/kg}$$

$$g = 9.8 \text{ m/s}^2$$

$$a \approx 6.4 \times 10^6 \text{ m}$$

(^ Earth's radius)

Estimate values for:

[\bar{v} velocities]

$$\bar{v}_t =$$

$$\bar{v}_b =$$

Spec. humidity q is units of **g/kg**
Use MKS system for calculations.

[\bar{q} : specific humidity]

$$\bar{q}_t =$$

$$\bar{q}_b =$$

φ : latitude

$$\varphi_{\text{avg}} =$$

1 millibar = 10^2 Pascals

p : pressure

$$\Delta p =$$

Tip: convert specific humidity to kg/kg

→ Express \mathcal{L} in Petawatts, $\text{PW} = 10^{15} \text{ W}$.

$$\text{Total Heat Flux} = \mathcal{H} + \mathcal{L}$$

Meridional transport of moisture

$$\mathcal{L} = \frac{L}{g} \times 2\pi a \cos \varphi \int_0^{p_s} \overline{vq} dp$$

Two layer
approximation

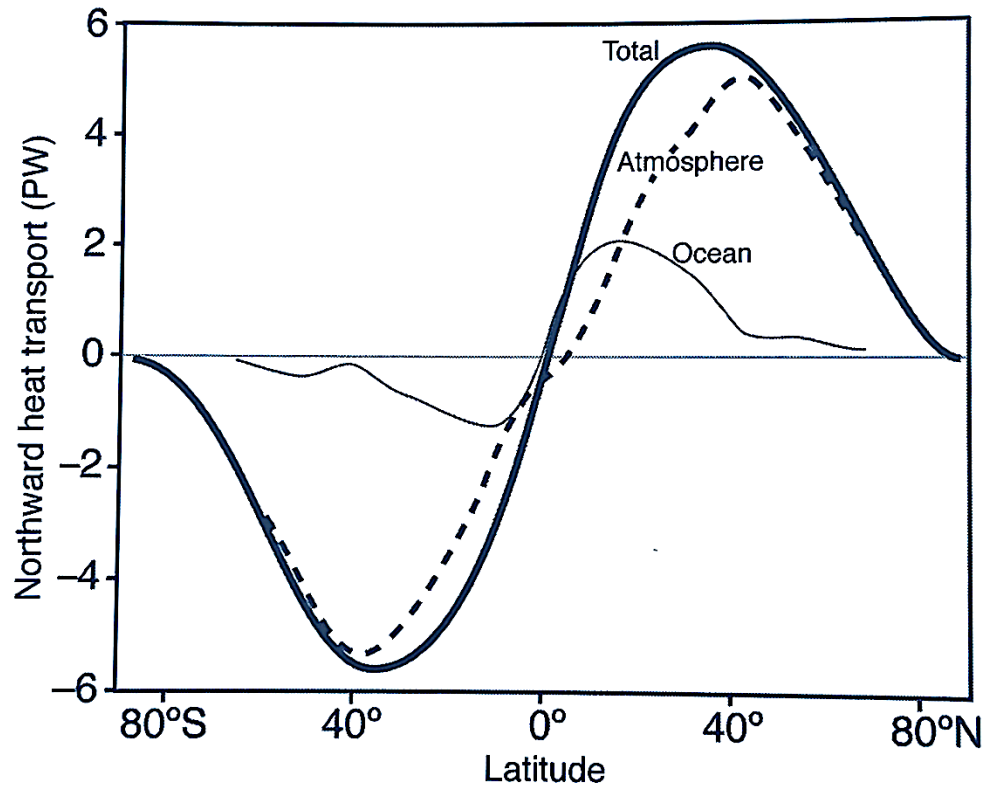
$$\simeq \frac{L}{g} \times 2\pi a \cos \varphi [\overline{v}]_b [\overline{q}]_b \Delta p.$$

Let's put some numbers, using the [schematic](#):

$$\begin{aligned} L &= [(2.25 \times 10^6)/10] \times (6 \times 6 \times 10^6) \times (-3 \times 15 \times 10^{-3}) \times (1.5 \times 10^4) \\ &= 5.4 \times 10^{15} \text{ W} = -5.4 \text{ PW} \end{aligned}$$

The moisture transport is negative (equatorward), since v_b is negative and moisture is larger near the surface!

Compare our estimates to reanalysis estimates:



Total heat transport = $H + L \sim 8 - 5.4 = 2.6$ PW
→ Not a bad rough estimation!