12.307: Project 2
Fronts in the atmosphere

Talia Tamarin-Brodsky and John Marshall

The air mass over the pole is considerably colder (and dryer) than that over the equator. The most active weather occurs in middle latitudes where the two air masses meet. The transition from cold to warm is not smooth, but occurs quite abruptly in a region of high gradients, known as the polar front. On the synoptic scale, fronts can be responsible for significant changes in the day-to-day weather, while the polar front plays an important part in dictating the pattern of the large-scale flow.

1 Slope of a frontal surface following Margules

The Margules (1906) relation used to investigate the front in the lab experiment can be also used to calculate the slope of a frontal surface in the atmosphere. Using the ideal gas equation, $p = \rho RT$, and the definition of potential temperature, $\theta = T(\frac{P_0}{P})^{\frac{R}{C_p}}$ (where $R = 287\ \frac{\mathrm{m}^2}{\mathrm{Ksec}^2}$ is the dry gas constant and C_p is the specific heat), the Margules relation can be written as

$$\tan\gamma = \frac{f(u_2 - u_1)}{g\frac{\theta_1-\theta_2}{\overline{\theta}}}, \tag{1}$$

where $f \approx 10^{-4}\ \frac{1}{\mathrm{sec}}$ is the Coriolis parameter, and u_1,u_2 & θ_1,θ_2 are the zonal wind speed and potential temperatures of the warmer (lighter) fluid ("1") and the colder (denser) fluid ("2"), respectively (following the same notation we used in the lab), and $\overline{\theta}$ can be taken as the average temperature.

1.1 Case study: The Polar Front of January 17, 2013

Here we are studying the polar front on January 17th, 2013 and make an analogy to the Front experiment we performed in class.

Fig.1 shows an instantaneous map of the temperature (in Celsius) at 500 mb over the northern hemisphere on the 17th of January, 2013 12Z. The air to the south is warm, that to the north is cold with a pronounced middle latitude temperature gradient - the so called polar front. Note a strong north-south temperature gradient over the midwest region of the US. The same instantaneous hemispheric map is shown in Fig.2, plotting potential temperature (in K) instead of temperature itself. Read here for further (optional) reading on potential temperature.

Fig.3 shows a north-south vertical section of potential temperature along the 80W longitude on January 17th, 2013 at 12z. Please note that the x-axis is latitude and the y-axis is the ln(p) (mb), which is proportional to height. The frontal zone between the cold and warm air has been marked by shading in green air with temperature below 295 K. To the north note the dome of cold/dense air (in green) with the warm/light air sliding on top of it to the south. The contours show a north-south

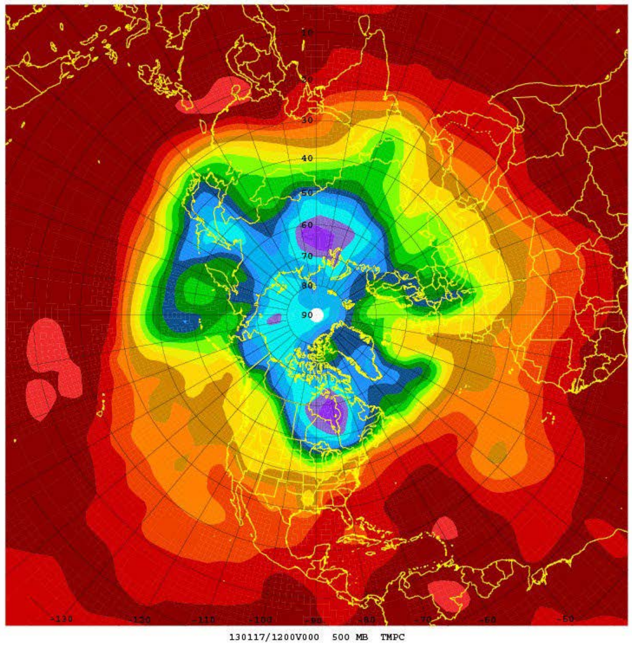

Figure 1: Temperature (°C) at 500 mb over the Northern Hemisphere on the 17th of January, 2013 12Z.

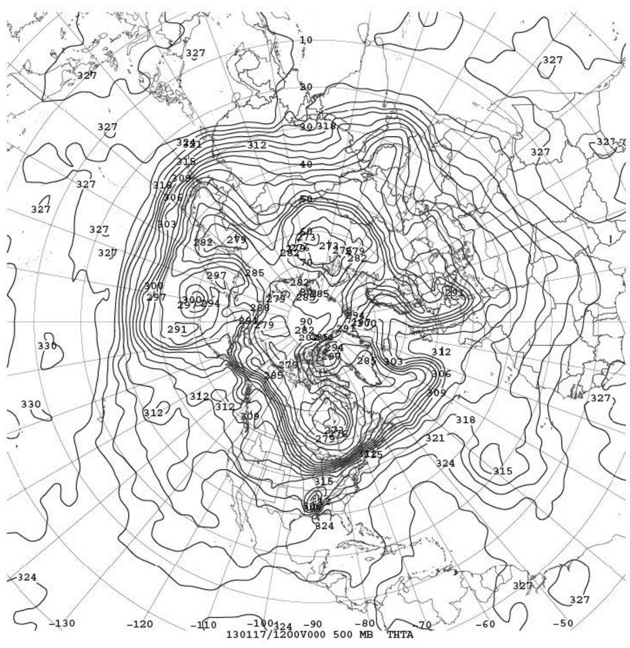

Figure 2: Potential temperature (K) at 500 mb over the Northern Hemisphere on the 17th of January, 2013 12Z.

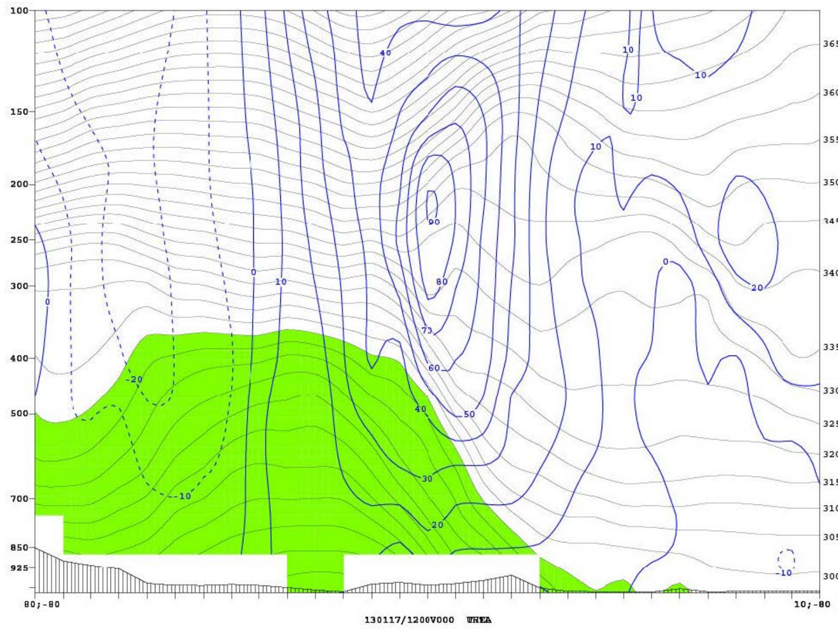

Figure 3: A north-south vertical section of potential temperature (in K, grey contours) and zonal wind (W-E) (in ms^{-1}, blue contours), along the 80W longitude on January 17th, 2013 at 12z. Green shading denotes potential temperature below 295 K.

section of the zonal wind (W-E) (in ms-1). Note that the wind increases rapidly with height with a maximum around 250 mb.

Use the Margules equation to estimate the slope of the frontal surface, and compare to a direct calculation of the slope by estimating the height and horizontal scale of the front. Does the Margules equation give a good prediction to the slope?

2 Lagrangian vs Eulerian derivative

Consider the situation sketched in Fig.1 in which a wind blows over a hill. The hill produces a pattern of waves in its lee. If the air is sufficiently saturated in water vapor, the vapor often condenses out to form cloud at the 'ridges' of the waves.

Let us suppose that a steady state is set up so the pattern of cloud does not change in time. If $C = C(x, y, z, t)$ is the cloud amount, where (x, y) are horizontal coordinates, z is the vertical coordinate, t is time, then:

$$\left(\frac{\partial C}{\partial t}\right)_{\substack{\text{fixed point} \\ \text{in space}}} = 0,$$

where we keep at a fixed point in space, but at which, because the air is moving, there are constantly changing fluid parcels. The derivative $\left(\frac{\partial}{\partial t}\right)_{\text{fixed point}}$ is called the 'Eulerian derivative' after Euler.

But C is not constant *following along a particular parcel*; as the parcel moves upwards into the ridges of the wave, it cools, water condenses out, cloud forms, and so C increases; as the parcel moves down into the troughs it warms, the water goes back in to the gaseous phase, the cloud disappears and C decreases. Thus

$$\left(\frac{\partial C}{\partial t}\right)_{\substack{\text{fixed} \\ \text{particle}}} \neq 0$$

even though the wave-pattern is fixed in space and constant in time.

So, how do we mathematically express 'differentiation following the motion'? In order to follow particles in a continuum a special type of differentiation is required. Arbitrarily small variations of $C(x, y, z, t)$, a function of position and time, are given to the first order by:

$$\delta C = \frac{\partial C}{\partial t}\delta t + \frac{\partial C}{\partial x}\delta x + \frac{\partial C}{\partial y}\delta y + \frac{\partial C}{\partial z}\delta z$$

where the partial derivatives $\frac{\partial}{\partial t}$ etc. are understood to imply that the other variables are kept fixed during the differentiation. The fluid velocity is the rate of change of position of the fluid element, following that element along. The variation of a property C *following an element of fluid* is thus derived by setting $\delta x = u\delta t$, $\delta y = v\delta t$, $\delta z = w\delta t$, where u is the speed in the x-direction, v is the speed in the y-direction and w is the speed in the z-direction, thus:

$$(\delta C)_{\substack{\text{fixed} \\ \text{particle}}} = \left(\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} + w\frac{\partial C}{\partial z}\right)\delta t$$

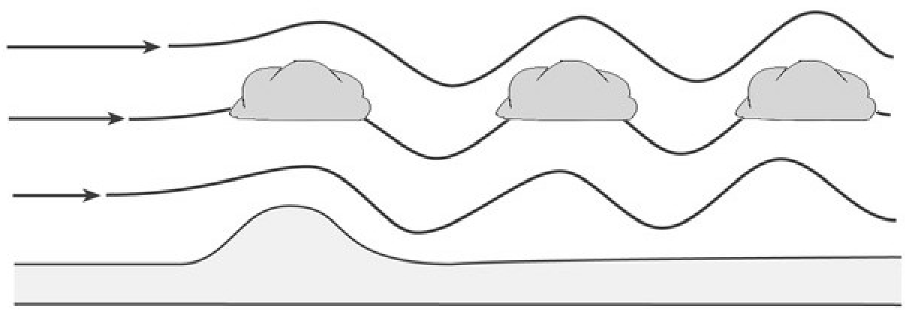

Figure 1: A schematic diagram illustrating the formation of mountain waves (also known as lee waves). The presence of the mountain disturbs the air flow and produces a train of downstream waves (cf., the analogous situation of water in a river flowing over a large submerged rock, producing a downstream surface wave train). Directly over the mountain, a distinct cloud type known as lenticular ("lens-like") cloud is frequently produced. Downstream and aloft, cloud bands may mark the parts of the wave train in which air has been uplifted (and thus cooled to saturation).

where (u, v, w) is the velocity of the material element which by definition is the fluid velocity. Dividing by δt and in the limit of small variations we see that:

$$\left(\frac{\partial C}{\partial t}\right)_{\substack{\text{fixed}\\\text{particle}}} = \underbrace{\frac{\partial C}{\partial t}}_{\text{fixed point}} + \underbrace{u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} + w\frac{\partial C}{\partial z}}_{\text{advection}} = \frac{DC}{Dt} \tag{1}$$

in which we use the symbol $\frac{D}{Dt}$ to identify the rate of change following the motion:

$$\frac{D}{Dt} \equiv \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z} \equiv \frac{\partial}{\partial t} + \mathbf{u}.\nabla \ . \tag{2}$$

Here $\mathbf{u} = (u, v, w)$ is the velocity vector and $\nabla \equiv \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$ is the gradient operator. Note that the second term in Eq.(1) is known as 'advection' and represents the property of a fluid to carry proporties along with it as it moves.

The expression $\frac{D}{Dt}$ is called the Lagrangian derivative (after Lagrange; 1736 - 1813) [it is also called variously the 'substantial', the 'total' or the 'material' derivative]. Its physical meaning is 'time rate of change of some characteristic of a particular element of fluid' (which in general is changing its position). By contrast, as introduced above, the Eulerian derivative $\frac{\partial}{\partial t}$, expresses the rate of change of some characteristic at a *fixed point* in space (but with constantly changing fluid element because the fluid is moving).

Examples:

- Velocity and trajectories.

  The position of the parcel of fluid is related to its velocity thus (in 2-dimensions):

$$u = \frac{D}{Dt}x; \quad v = \frac{D}{Dt}y \tag{3}$$

$$x = \int u dt; \; y = \int v dt \tag{4}$$

where u is the speed in the x direction and v is the speed in the y direction etc.

- Tracer transport.

  If a tracer concentration of a parcel of fluid, T say, does not change as it moves along, then:

  $$\frac{D}{Dt}T = 0. \tag{5}$$

  This is clearly illustrated in the dye-stir experiment where fluid parcels conserve (but for small diffusive processes) the concentration of dye that marked parcels when the dye was injected. Over time the fluid parcels intermingle drawing the colors they carry in to close proximity to form the beautiful swirling patterns.

- Familiar meteorological example.

  Figure 2 shows an instantaneous map of the northern hemisphere temperature at 850mb (roughly 2 km above the surface) on a day in January. The red shows warm air in the tropics, the blue cold air of polar latitudes. In regions where the cold (blue) air is moving south ($v < 0$), the local rate of change of temperature is[1]:

  $$\frac{\partial T}{\partial t} \simeq -v\frac{\partial T}{\partial y} < 0$$

  because $v < 0$ and $\frac{\partial T}{\partial y} < 0$. Thus at a particular point on the Earth (e.g. Boston), the temperature decreases. The reverse happens when warm (red) air moves north. This is the process known as 'advection'.

3 Fluid transport in the Atmosphere

3.1 Calculating trajectories from wind observations

We learned in Section 1.1 how to express the rate of change of a property C of a fluid element, *following that element as it moves along*, rather than at a fixed point in space — see Eq.(1)

[1] Here y increases north and the meridional wind speed is $v = Dy/Dt$ and is positive if the wind is blowing north. The meridional temperature gradient is $\partial T/\partial y < 0$ if it gets colder moving north.

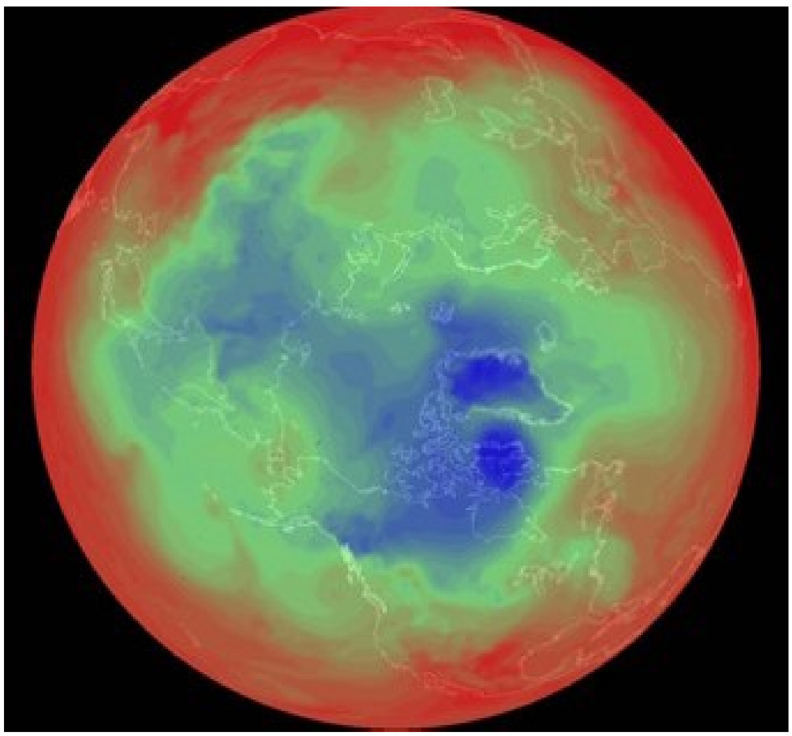

Figure 2: Instantaneous map of the northern hemisphere temperature at 850mb (roughly 2 km above the surface) on a day in January. Warm is red and cold is blue. You can view a movie loop here: http://weathertank.mit.edu/wp-content/uploads/2017/04/850mbPotTempLOOP.gif

and attendant discussion. It follows from the definition of the Lagrangian derivative, D/Dt, that the position of a parcel of fluid is related to its velocity by Eqs(3 and 4, repeated here for 2-dimensions:

$$u = \frac{D}{Dt}x; \;\; v = \frac{D}{Dt}y,$$

$$x = \int udt; y = \int vdt,$$

where u is the speed in the x direction and v is the speed in the y direction etc.

Here we will build the trajectory — (x, y) as a function of time — of a hypothetical particle of dust, moving with the wind during June of 2018 when satellite imagery revealed a persistent flow of dust from the Sahara desert moving across the Atlantic over the Southern US. In fact the last 10 days of June were the 10 dustiest for the tropical Atlantic going back 15 years — see Fig.3 — from NASA-Earth Observatory — see https://earthobservatory.nasa.gov/images/92358/here-comes-the-saharan-dust.

Using maps (handed out in class) of the height field and the wind field at the 850 mb level over the Atlantic region at: 180620 (20th June, 2018), 12 GMT; 180620 18 GMT; 180621 00 GMT and 180621 06 GMT, draw by hand the trajectories of chosen particles following the 12z wind for the first six hours, the 18z wind for the second six hours, the 00z wind for the third six hours and the 06z wind for the fourth six hours. In this way, compute a trajectory over a period of 1 day.

Compare your trajectory with one you obtain from the EsGlobe interface by launching a virtual particle in to the same flow field. The two should be rather similar. This is essentially how the EsGlobe computes trajectories: it reads in wind fields at regular intervals and performs the above integrals numerically, plotting out the positions as it goes.

Finally, estimate how long it will take for your particle of dust to reach Texas. Is your estimate consistent with what was observed by satellite, as described in the article above?

3.1.1 Rate of change of temperature at a point: temperature advection

More often than not, it becomes cold over Boston, say, because winds carry low temperatures from places where it is cold. We say that cold air is advected (carried) by the winds. We can write this down mathematically as:

$$\frac{D}{Dt}T = 0, \tag{6}$$

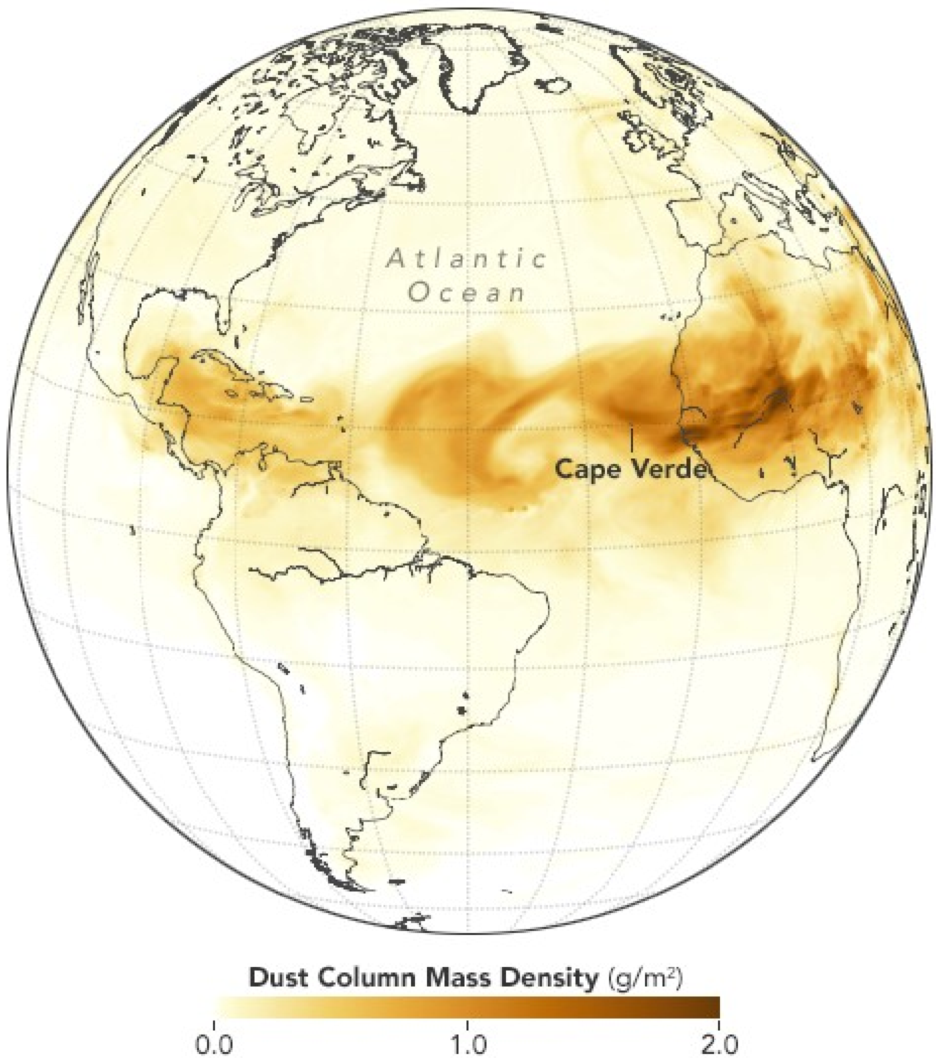

Figure 3: A cloud of dust, whipped up winds from the Sahara Desert, being carried by the trade winds over to the Caribbean, and on toward Texas. The map is from June 28, 2018: from NASA-Earth Observatory — see https://earthobservatory.nasa.gov/images/92358/here-comes-the-saharan-dust.

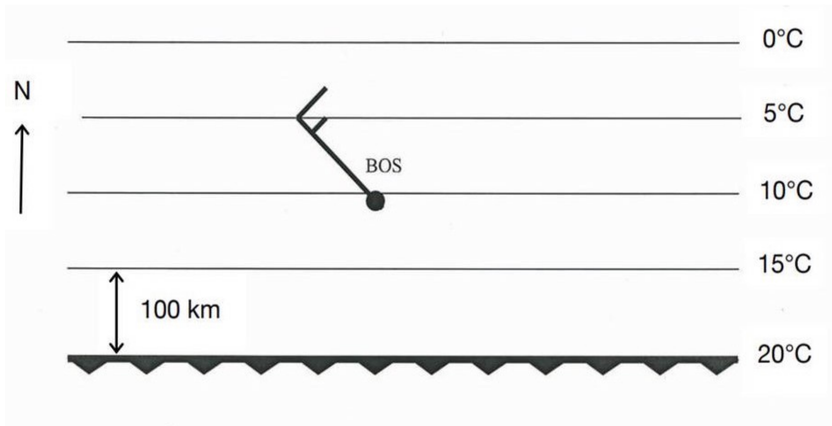

Figure 4: Sketch of a front (indicated by the thick serrated line) oriented west to east across which the temperature drops 5^oC every 100 km. The wind is blowing from the NW at $15kts$.

assuming that air parcels conserve temperature as they move. If the winds are predominantly horizontal (a good approximation) then:

$$(\partial T/\partial t)_{\text{at Boston}} = -u\partial T/\partial x - v\partial T/\partial y \tag{7}$$

where u is the zonal (west to east) component of the wind and v is the meridional (south to north) component of the wind, T is temperature and t is time.

If the temperature at Boston increases with time:

- then $\partial T/\partial t > 0$ and is associated with warm temperature advection (winds blowing from higher to lower temperature)

Vice versa, if the temperature at Boston decreases with time:

- then $\partial T/\partial t < 0$ and is associated with cold temperature advection (winds blowing from lower to higher temperature)

Let's test these ideas out using data by comparing the local temperature tendency at a few locations to that implied by horizontal temperature advection, as expressed in Eq.(7).

A Schematic front. Suppose a cold front has just passed over Boston. The front is oriented west to east and the temperature drops 5^oC every 100 km (as sketched in Fig.4). As the wind blows from the NW at $15kts$, where $1kts = 0.5m/s$, infer how much the temperature will be expected to drop in 12 hours due to cold air advection?

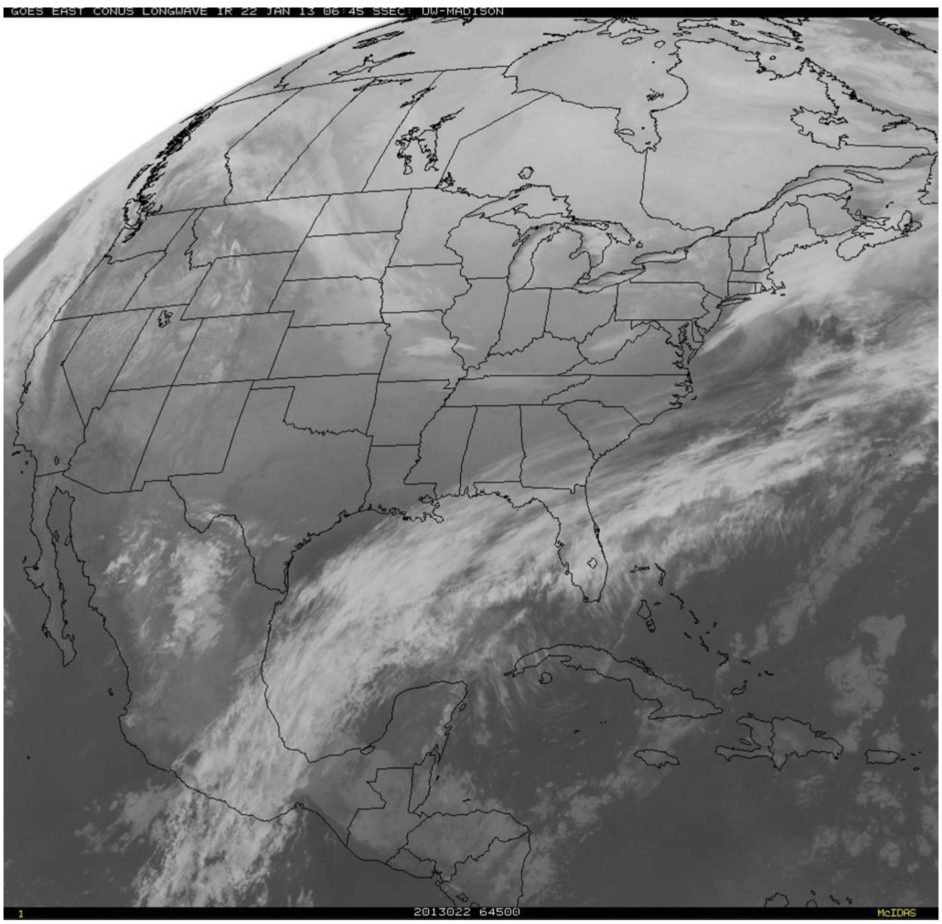

Figure 5: IR satellite image for January 22, 2013 at 06z

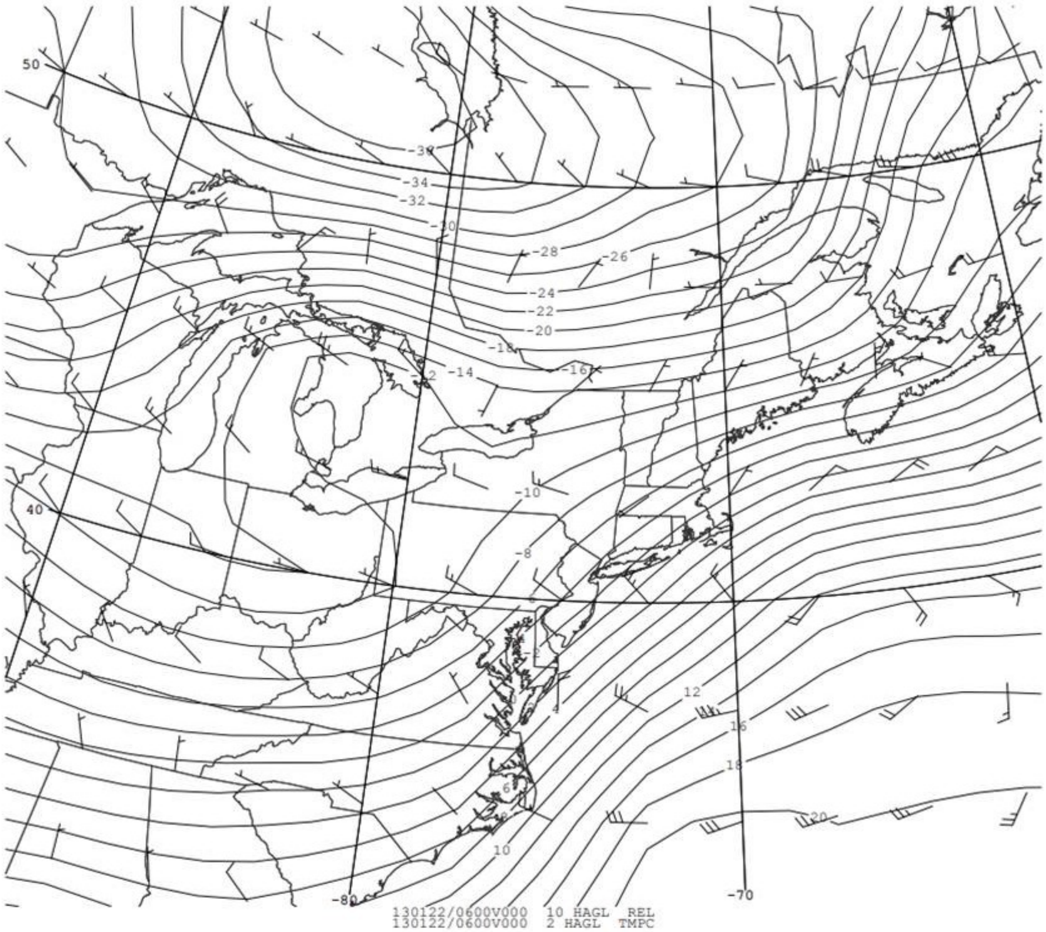

Figure 6: Analyzed surface temperature (contored in ^oC) and surface wind (vectors in kts) for the same time, as in January 22, 2013 at 06z.

A 'Real' front. Fig.5 is an IR satellite image for January 22, 2013 at 06z, showing a band of clouds, associated with a pronounced northerly flow along the east coast of the US.

Fig.6 shows analyzed surface temperature (in ^oC) and surface wind (in kts) for the same time, January 22, 2013 at 06z. Estimate the horizontal temperature advection at Chicago-O'Hare, IL (ORD) and Pittsburgh, PA (PIT). What is the expected 6-hour temperature change due to this horizontal temperature advection?

Compare your results with the observed temperature changes revealed in the surface meteograms shown in Figs.7 and Fig.8.

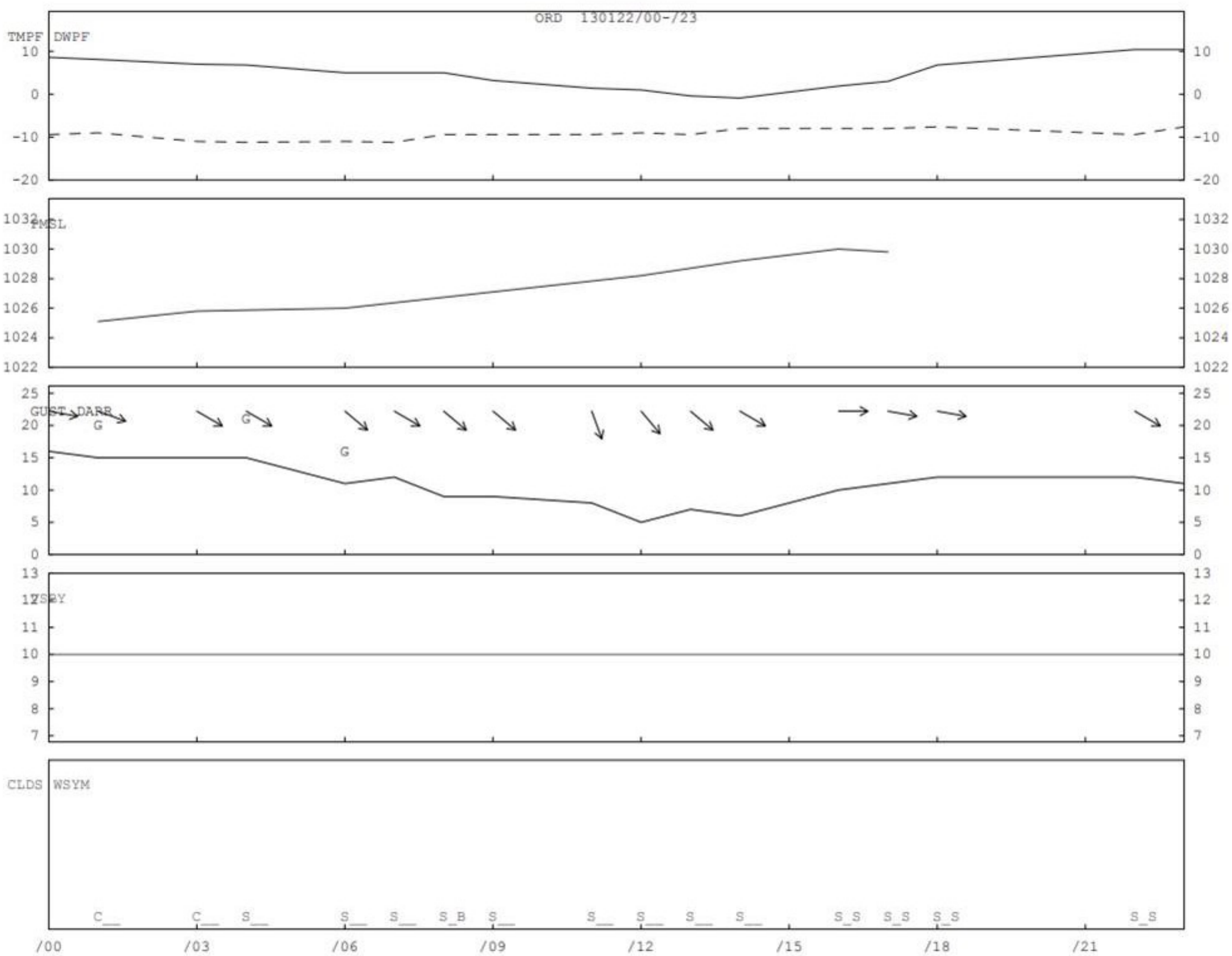

Figure 7: Surface meteogram for Chicago O'Hare (ORD) on January 22, 2013 showing temperature (continuous line) and dewpoint temperature (dashed line), surface pressure, wind speed and direction, visibility and cloud cover.

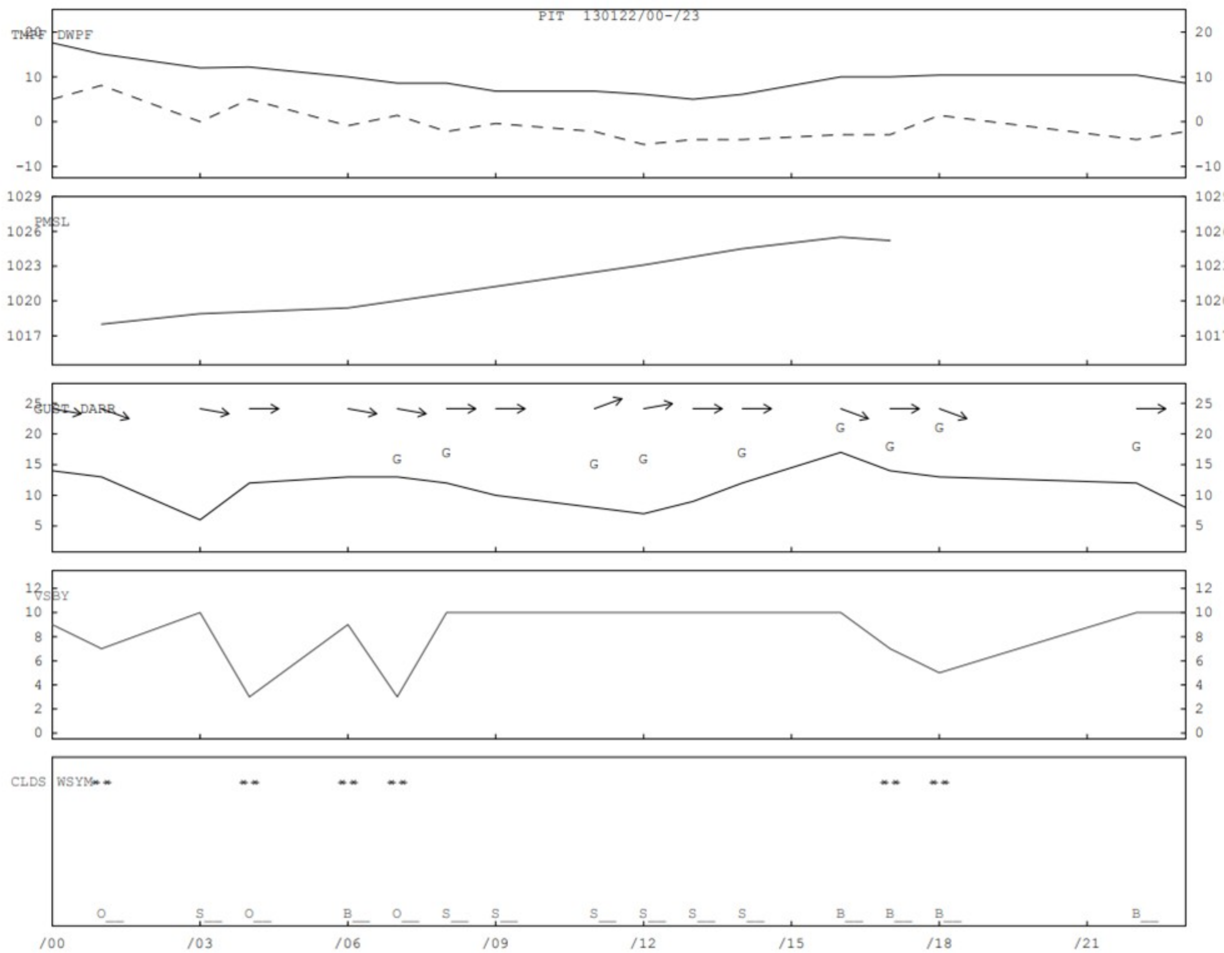

Figure 8: Surface meteogram for Pittsburg (PIT) on on January 22, 2013 showing temperature (continuous line) and dewpoint temperature (dashed line), surface pressure, wind speed and direction, visibility and cloud cover.

4 Temperature Advection and Variance

In the previous section, we examined the climatological mean temperature structure. We can define temperature anomalies T' as deviation from that climatology, i.e.:

$$T' = T - T_{\text{clim}}, \tag{2}$$

where $T_{\text{clim}} = \overline{T}$ is the averaged temperature.

The temperature anomalies that we experience on a daily basis are closely related to passing cyclones and anticyclones, and similarly have typical timescales of a few days.

In a climate sense, we can quantify the strength of fluctuations around the average temperature using the *variance* ($\overline{T'^2}$), the average of the square of T' (note that by definition, $\overline{T'} = \overline{T - T_{\text{clim}}} = \overline{T} - T_{\text{clim}} = 0$). The temperature variance is a measure of the spread of temperatures around the mean.

Considering the Lagrangian derivative of temperature, and assuming that it is nearly conserved (which is not generally correct), we can write-

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = 0. \tag{3}$$

Assuming further that:

$$T' << \overline{T}, \text{ temperature anomalies are smaller than the mean temperature,} \tag{4}$$

$$\frac{\partial \overline{T}}{\partial x} << \frac{\partial \overline{T}}{\partial y}, \text{ meridional temperature gradient is larger than the zonal gradient,} \tag{5}$$

$$\overline{v} << \overline{U}, \text{ is the mean meridional velocity is smaller than the mean zonal velocity,} \tag{6}$$

Eq. (5) reduces to

$$\frac{\partial T'}{\partial t} \approx -v'\frac{\partial \overline{T}}{\partial y}. \tag{7}$$

Noting further that $v' = \frac{d\eta'}{dt}$, where η' is the displacement of the air parcel, and approximating $v' \approx \frac{\partial \eta'}{\partial t}$, we can write

$$T' \approx -\eta'\frac{\partial \overline{T}}{\partial y}. \tag{8}$$

Thus, temperature variance is proportional to the square of the meridional temperature gradient,

$$\overline{T'^2} \sim (\frac{\partial \overline{T}}{\partial y})^2. \tag{9}$$

4.0.1 Temperature-Variance Exercise:

1) How justified are the assumptions we made in deriving the relation given in (11)? Can you verify the assumptions given in (7) and (8) using the EsGlobe?

2) We will be using data of near-surface temperature (T_{2m}, the temperature 2-meters above the ground) from 7 state-of-the-art global circulation models, which are used in current research to study our climate and its projected changes. The data cover the period of 1980-2014 for the historical simulations, and for 2065-2099 for the projected climate. The data you will be working with is based on 3-hourly near-surface temperature data, downloaded from https://aims2.llnl.gov/search/cmip6/. It is pre-processed such that it contains the mean temperature and the temperature variance as a function of longitude, latitude, year, and model. [1],

Go to the course website (2nd project, Observation Data page) and download the zip folder "temperature_variance". Unzip the files and put them in the same folder. Run the file plot_T2m.m (in MATLAB) or plot_T2m_python.py (in python). This should produce a figure showing the historical mean T_{2m} data for one model in the first data year. Now, modify the script so that it calculates the mean over all models and all years, and plot the historical mean T_{2m}, historical T_{2m} variance, and their projected changes. There are some further instructions on the script.

3) Questions:

- What do you find for the T_{2m} mean temperature and variance in the historical simulations? Is it similar to what we saw in class for the 850mb level?
- What do the projected mean temperature and variance show? Can you explain this response using temperature advection arguments?
- **Optional**: Examine the model-to-model spread and the year-to-year variability of global mean temperature. Do all models agree on the changes? Can you observe a trend in the historical/projected data? Is the trend larger than the year-to-year variability?

[1] The models used are: CMCC-CM2-SR5, CanESM5, MIROC-ES2L, MIROC6, MPI-ESM1-2-HR, MPI-ESM1-2-LR, and MRI-ESM2-0