2 Part I: Laboratory Experiment: Cylinder collapse

You can read about the set up of our laboratory experiment here — http://weathertank.mit.edu/links/projects/fronts-an-introduction — where associated theory and recordings of previous experiments can also be found. The apparatus is illustrated in Fig.4.

In our first class we demonstrated two experiments, one non-rotating the other rotating. You will now carry out the rotating experiment yourself, collect data and interpret it in terms of the Margules theory presented in section 2.1 below. This same theory, appropriately modified for application to a compressible atmosphere, will be used to interpret the structure of atmospheric fronts in part II.

The column of dense salty water slumps under gravity but is 'held up' by rotation forming a cone whose sides have a distinct slope. Because of the spin imparted to the fluid by the rotation of the table, the collapsing column must satisfy an angular momentum constraint. Its final state is not the intuitive one, with resting light fluid over dense separated by a horizontal interface. Instead the collapsed column remains tilted and the fluid contained within it is in horizontal swirling motion having 'concentrated' (at the top) and 'diluted' (at the bottom) the angular momentum of the rotating table — an example is given in Fig.5. The action of gravity trying to make the interface horizontal is balanced by the (difference in) centrifugal forces on the swirling currents induced by angular momentum.

Our goal is to quantify the connection between the tilt of the interface separating salty and fresh water to the density difference, the vertical shear of the currents across the front and the rotation rate of the turntable. To provide a framework we will now derive some background theory.

2.1 Background theory following Margules

A simple and instructive model of a front can be constructed as follows, following Margules (1906). Suppose that at some height z the density is ρ_2 on one side of the front and then changes discontinuously to ρ_1 on the other [see Fig.(6)]. Let x be a horizontal axis normal to the discontinuity and let γ be the angle that the surface of discontinuity makes with the horizontal.

1. Since the pressure must be the same on both sides of the discontinuity,

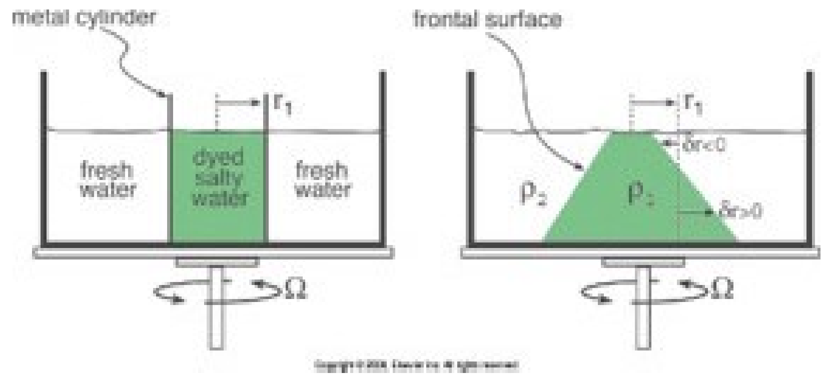

Figure 4: (left) The initial state of the cylinder collapse experiment, before removal of the metal cylinder. (right) The quasi-equilibrium state after removal of the cylinder in which the dense cone of fluid is in thermal wind balance with a well defined slope.

use the hydrostatic relation, $\frac{\partial p}{\partial z} + \rho g = 0$, to show that:

$$\tan\gamma = \frac{dz}{dx} = \frac{\frac{\partial p_1}{\partial x} - \frac{\partial p_2}{\partial x}}{g\left(\rho_1 - \rho_2\right)} \tag{1}$$

where ρ_2 is the density of the ambient fluid and ρ_1 is the density within the cone (and $\rho_1 > \rho_2$).

2. Using the geostrophic approximation to the current ($fv_1 = \frac{1}{\rho_1}\frac{\partial p_1}{\partial x}$, $fv_2 = \frac{1}{\rho_2}\frac{\partial p_2}{\partial x}$) and noting that $\frac{\rho_2}{\rho_1} \simeq 1$, show that:

$$v_2 - v_1 = \frac{g' \tan\gamma}{2\Omega} \tag{2}$$

where v is the component of the current parallel to the front,

$$g' = g\frac{(\rho_1 - \rho_2)}{\rho_1} \tag{3}$$

is the reduced gravity and Ω is the rotation rate of the tank.

We can use Eq.(2) to predict the slope of the sides of the cone observed in the laboratory experiments from the change in density between the cone and the ambient fluid, $\frac{(\rho_1 - \rho_2)}{\rho_1}$, and the measured swirl velocities.

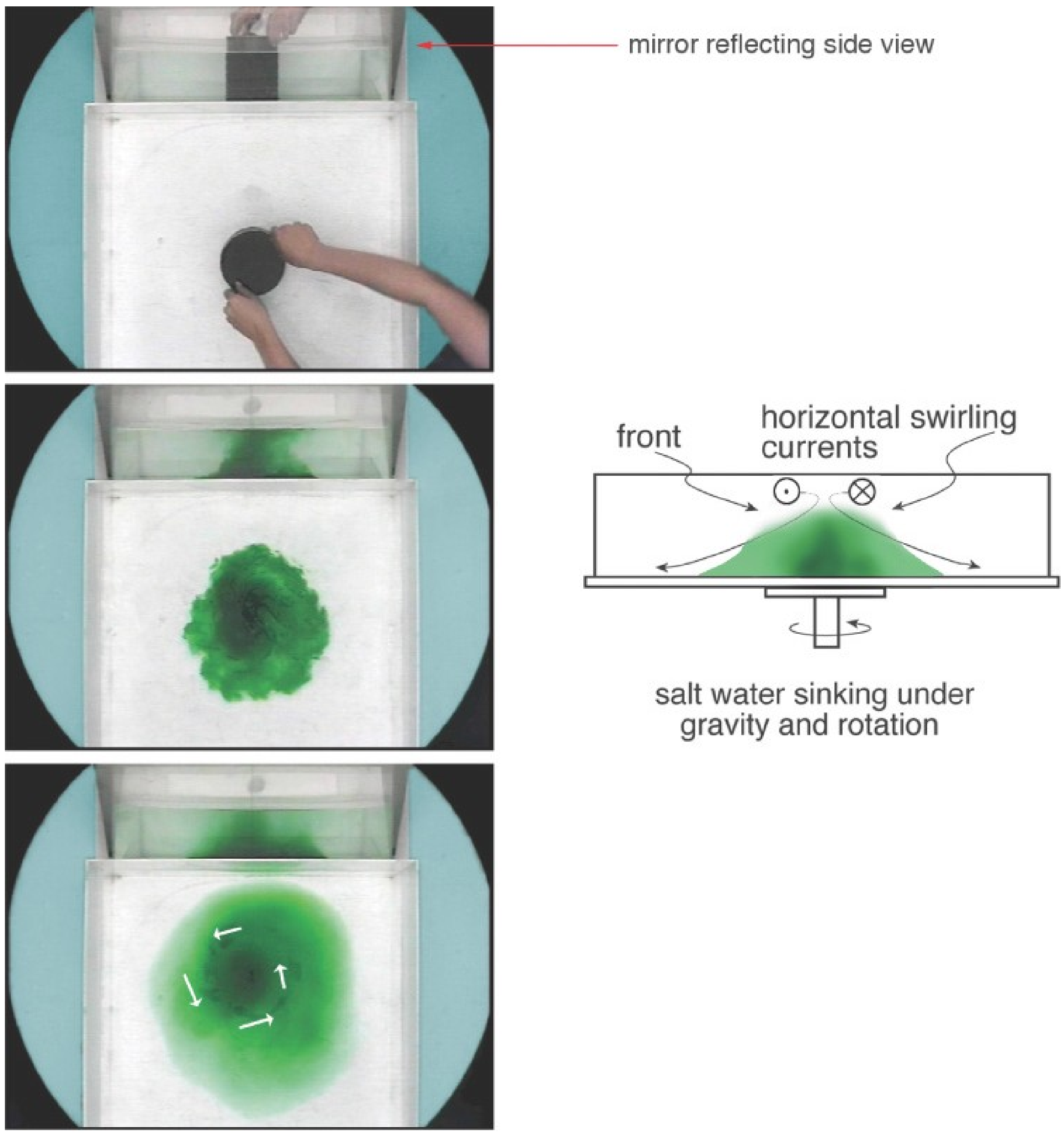

Figure 5: Left: Series of pictures charting the creation of a dome of salty (and hence dense) dyed fluid collapsing under gravity and rotation. The fluid depth is 10cm. The white arrows indicate the sense of rotation of the dome. At the top of the figure we show a view through the side of the tank facilitated by a sloping mirror. Right: A schematic diagram of the dome showing its sense of circulation. From Marshall and Plumb, 2008.

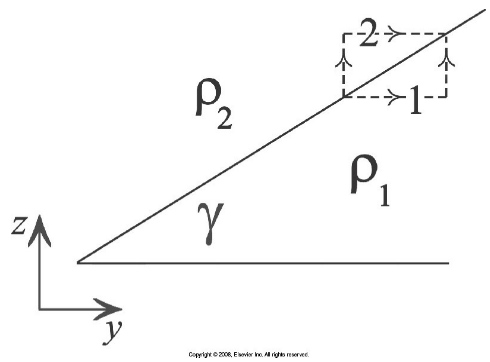

Figure 6: Geometry of the front separating fluid of differing densities used in the derivation of the Margules relation, Eq.(2).

2.2 Measurements

We measure:

1. the density of the dyed water and the clear water for evaluation of the reduced gravity g', Eq.(3). This can be done by weighing a known volume of a sample of the water, or by using a refractometer.[2]

2. typical surface speeds, v_2, above the cone at specific points making use of paper dots. Three particles seem to do the trick – two placed at different locations over the spinning cone of dense water and one placed in the far field as a reference. Use the particle tracker to help quantify your results.

3. typical bottom speeds, v_1, through the (parsimonious) use of potassium permanganate crystals[3].

[2] A refractometer is an optical device used to measure the density or concentration of a liquid, such as water, by observing how light bends (refracts) when passing through a small sample placed on a glass prism or panel. You look through an eyepiece to see a scale that indicates the refractive index, which is correlated with density.

[3] Potassium permanganate crystals are often used as a tracer to track water motion because they dissolve readily in water and release a vivid purple color, making it easy to visualize flow patterns, currents, or dispersion. You drop a few crystals into the water and watch how the colored trails spread. The crystals sink initially due to their density (about

4. the slope of the side of the cone (the frontal surface), γ, by taking a picture that captures the front and surface paper dots when they are aligned in a plane that is parallel to the camera.

How large is the Rossby number? Is it small enough that we can make the geostrophic approximation?
Interpret your observations in terms of the theory — Eq.(2) above.

2.7 g/cm^3, much heavier than water's 1 g/cm^3), then dissolve, releasing the permanganate ions that stain the water. This method is simple and effective for qualitative observation, though care must be taken to make it precise.