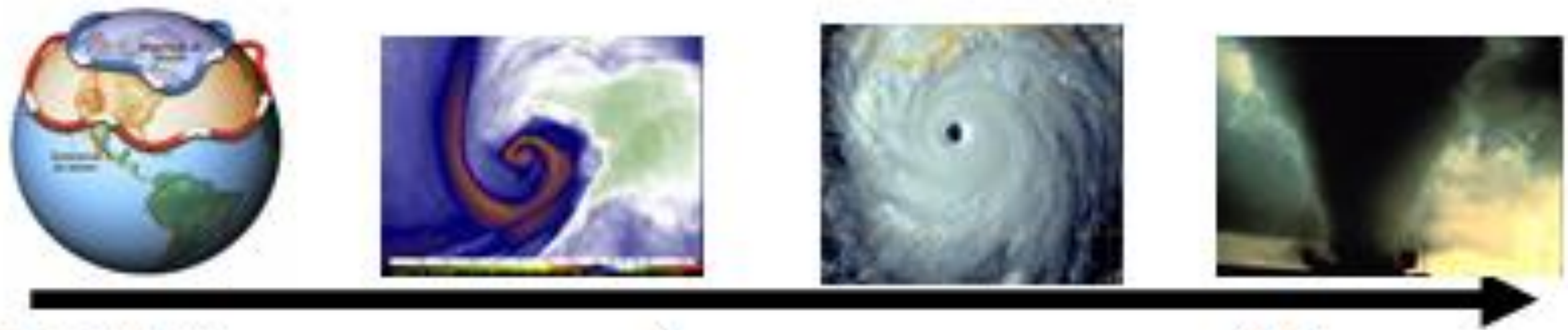
P1: Vortices in the atmosphere

http://weatherclimatelab.mit.edu/projects/weather-and-extremes/observation-data



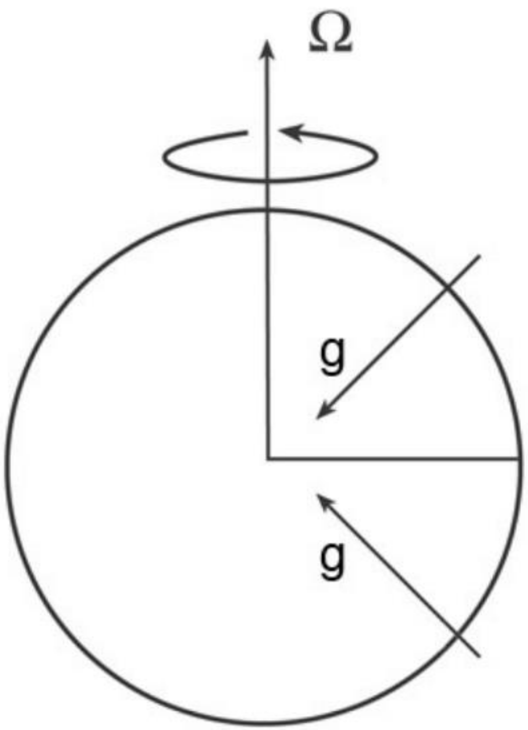
jet stream blizzard hurricane tornado

Today:

- Geostrophic balance on a sphere
- Measurement errors (Robert)
- Hurricane data (scatterometer)
- Quick guidelines for the presentation
- Jane Abbott (CIM presentations)- at 4pm in room 1623

Rotation on a sphere

In the rotating frame: $\mathbf{u}_{in} = \mathbf{u}_{rot} + \mathbf{\Omega} \times \mathbf{r}$



Sphere

$$\frac{D\mathbf{u}}{Dt} + \frac{1}{\rho}\nabla p + g\widehat{\mathbf{z}} = \underbrace{-2\Omega \times \mathbf{u}}_{\substack{\text{Coriolis} \\ \text{accel}^n}} + \underbrace{-\mathbf{\Omega} \times \mathbf{\Omega} \times \mathbf{r}}_{\substack{\text{Centrifugal} \\ \text{accel}^n}} + \mathcal{F}$$

For a parcel at rest in the rotating frame:

$$\frac{1}{\rho}\nabla p = -g\widehat{\mathbf{z}} - \mathbf{\Omega} \times \mathbf{\Omega} \times \mathbf{r}$$

The centrifugal term modifies gravity!

Rotation on a sphere

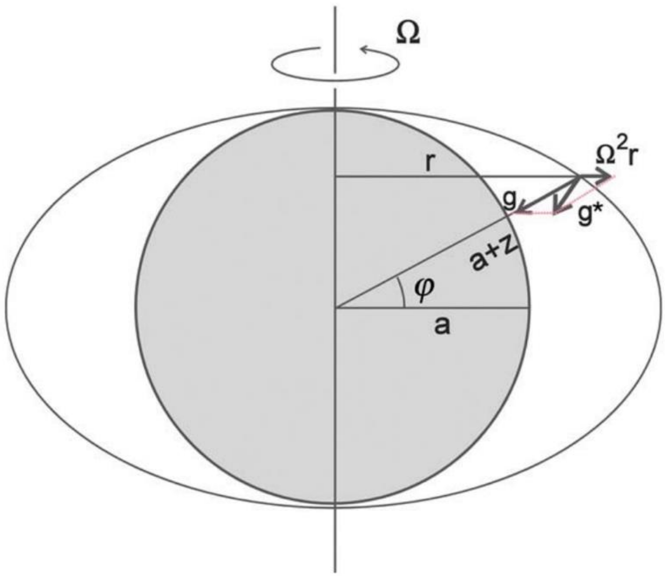


Figure 6.18: The centrifugal vector $\Omega \times \Omega \times r$ has magnitude $\Omega^2 r$, directed outward normal to the rotation axis. Gravity, g, points radially inwards to the center of the Earth. Over geological time the surface of the Earth adjusts to make itself an equipotential surface — close to a reference ellipsoid — which is always perpendicular the the vector sum of $\mathbf{\Omega} \times \mathbf{\Omega} \times \mathbf{r}$ and $\mathbf{g}$. This vector sum is 'measured' gravity: $\mathbf{g}^* = -g\widehat{\mathbf{z}} - \mathbf{\Omega} \times \mathbf{\Omega} \times \mathbf{r}$.

The modified gravitational potential on the Earth:

$$\phi = gz - \frac{\Omega^2 a^2 \cos^2 \varphi}{2}$$

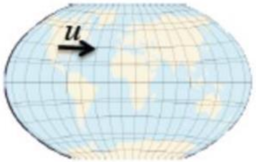
This is why Earth's surface is not actually a perfect sphere!

Since the second term on the RHS ≈ 11 km, geopotential surfaces depart only very slightly from a sphere, being 11 km higher at the equator than at the pole

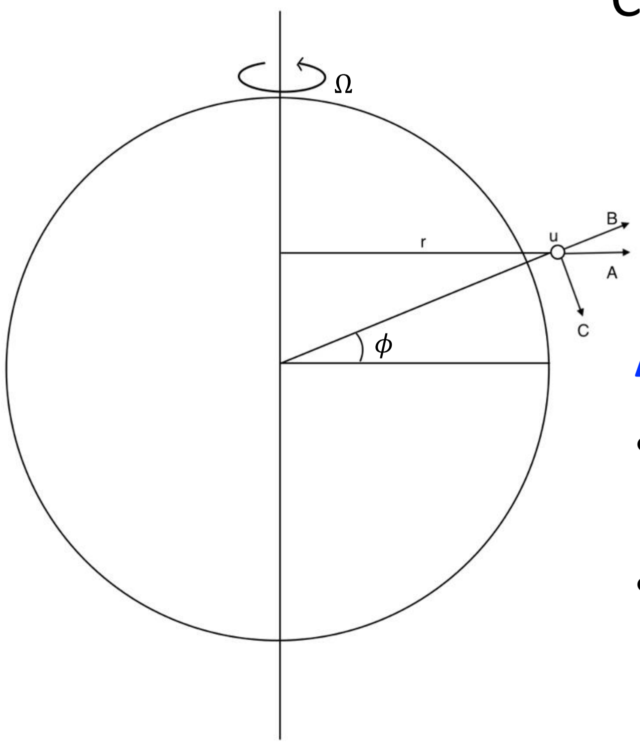
*** in reality, the difference is closer to 20 km, due to mass distribution around the equator*

Rotation on a sphere

Now consider a parcel moving from West to East with velocity U



Centrifugal Acceleration:



$$A = \frac{V^2}{r} = \frac{(U + \Omega r)^2}{r} = \frac{U^2}{r} + 2\Omega U + \Omega^2 r$$

Small compared to Cor if R0<<1

Included in g

A can be resolved into two components, B and C:

- B is perpendicular to the Earth's Surface and changes the weight of the ring slightly
- C is parallel to the Earth's Surface:

$$C \approx 2\Omega \sin\phi \, u = fu$$

is the Coriolis acceleration, and $\mathrm{f} = 2\Omega \sin\phi$ is the Coriolis parameter

The direction of the acceleration depends on the sign of u! (zero if at rest)

Coriolis Force

Facts about Coriolis force:

$$F_c = 2\Omega \sin\phi \, u = fu$$

- The Coriolis force causes objects to turn to the right of their direction of motion in the Northern hemisphere, and to the left in the Southern hemisphere!
- It can affect the direction of motion but not the speed!
- The magnitude of Coriolis force depends on:

(1) the rotation of the Earth

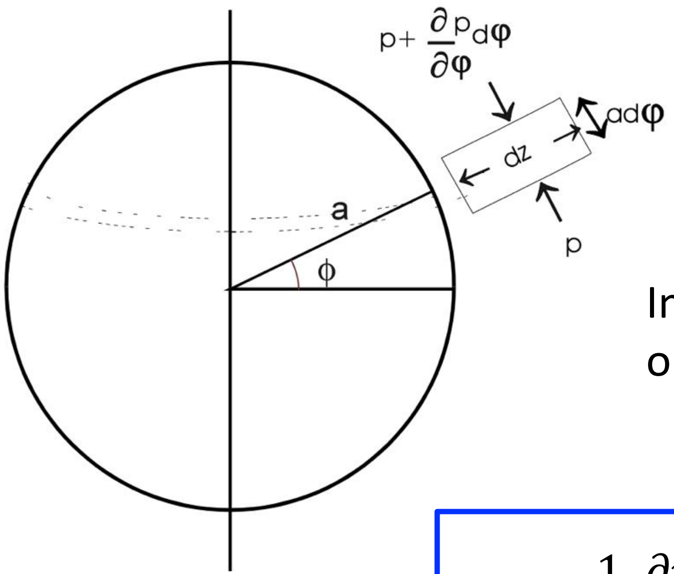
(2) the speed of the moving object (strongest for fast moving objects and zero at rest)

(3) its latitudinal location (is zero at the equator and maximum at the poles)

- The Coriolis force is most important for air movement over large distances and becomes insignificant at small scales

Coriolis force is very important process and shapes weather patterns

Geostrophic balance on the sphere



If Coriolis and PGF are in balance-

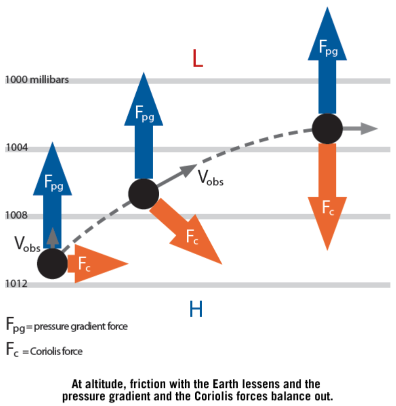
$$\underbrace{\rho a d\varphi dz}_{mass} \times \underbrace{2\Omega \sin\varphi u}_{acceleration} = \underbrace{-\frac{\partial p}{\partial \varphi} d\varphi dz}_{p\_grad}$$

Introducing a coordinate y which points northwards on the earth's surface- $dy = a d\varphi$

$$u = \frac{1}{\rho f}\frac{\partial p}{\partial y}$$

$$v = \frac{1}{\rho f}\frac{\partial p}{\partial x}$$

This is the geostrophic wind resulting from the balance between the PGF and the Coriolis force



where $f = 2\Omega \sin\phi$ is the Coriolis parameter

Geostrophic balance with friction

At the surface, friction plays an important role:

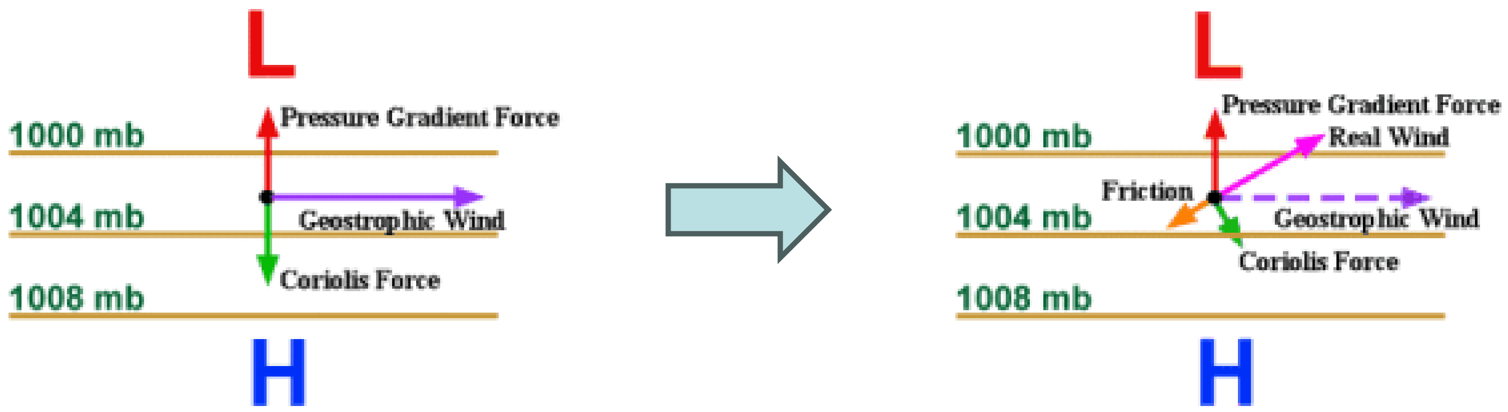


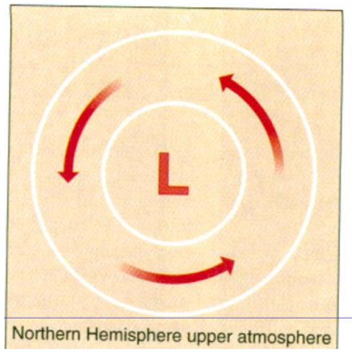
Figure from: http://ww2010.atmos.uiuc.edu/(Gh)/guides/mtr/fw/fric.rxml

Surface friction slows down the parcel → PGF "wins"

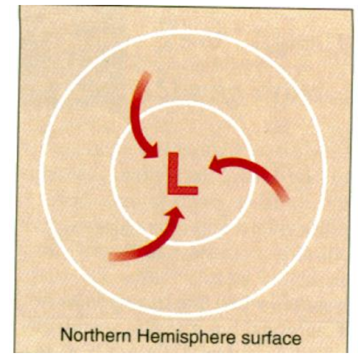
The flow is generally away from the high and towards the low

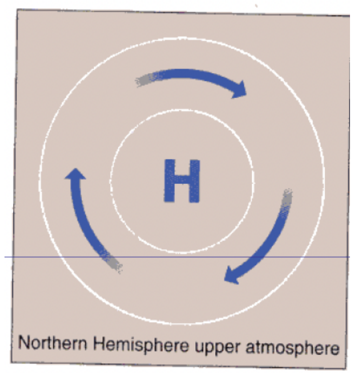
Geostrophic balance with friction

At the surface, friction plays an important role:

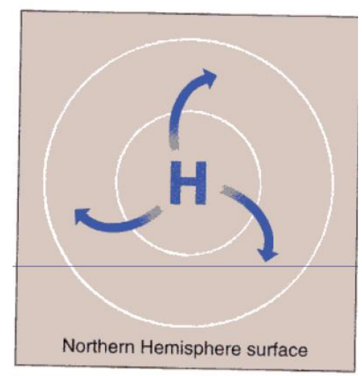


cyclones





Anticyclones



The flow is slightly away from the high and towards the low

Hurricanes

Radial force balance for atmospheric vortices

The radial force balance in atmospheric vortices is very similar to the radial force balance in the radial inflow experiment:

$$\frac{v_\theta^2}{r} + f v_\theta = \frac{1}{\rho}\frac{\partial p}{\partial r}$$

The main difference from the radial inflow experiment is that the Coriolis parameter is given by:

$$f = 2\Omega \sin \phi$$

Rossby number: As for the radial inflow experiment, we can use the radial force balance to define a Rossby number

$$Ro = \frac{|v_\theta|}{fr}$$

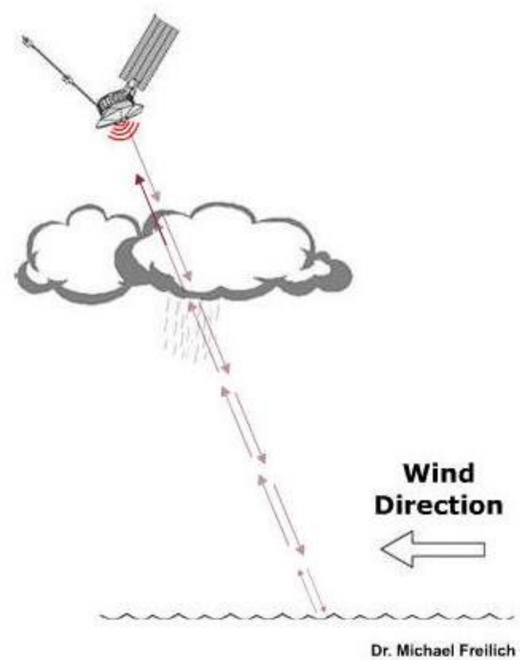
To study the balance of forces in a hurricane we are using a special dataset: surface wind data from the "**scatterometer**" instrument

See scatterometer_instructions

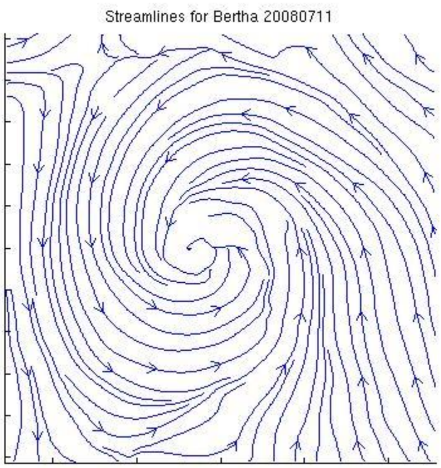
Scatterometer data to analyze Hurricanes!

Scatterometers are remote sensing instruments for deriving wind direction and speed from the roughness of the sea. They are used by low Earth orbiting satellites and act like radars: they transmit electromagnetic pulses and detect the backscattered signals

- Satellite sends down pulses of microwave radiation at ~5 GHz (European) or ~13-14 GHz (US)
- **Satellite very accurately measures the backscattered energy from the small-scale roughness elements on the ocean surface to determine wind speed and direction**



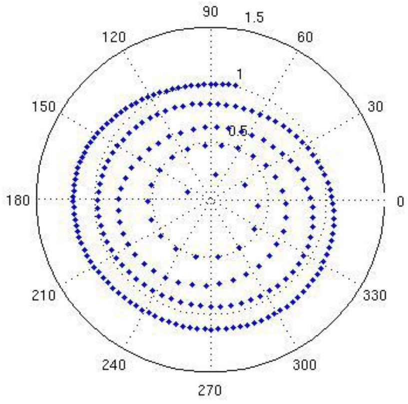
Streamlines:



Lines which smoothly connect the velocity vectors at an instance in time

- Shows the direction in which a fluid element will travel in an instantaneous flow
- Can change with time (unless the flow is steady)

Trajectories:

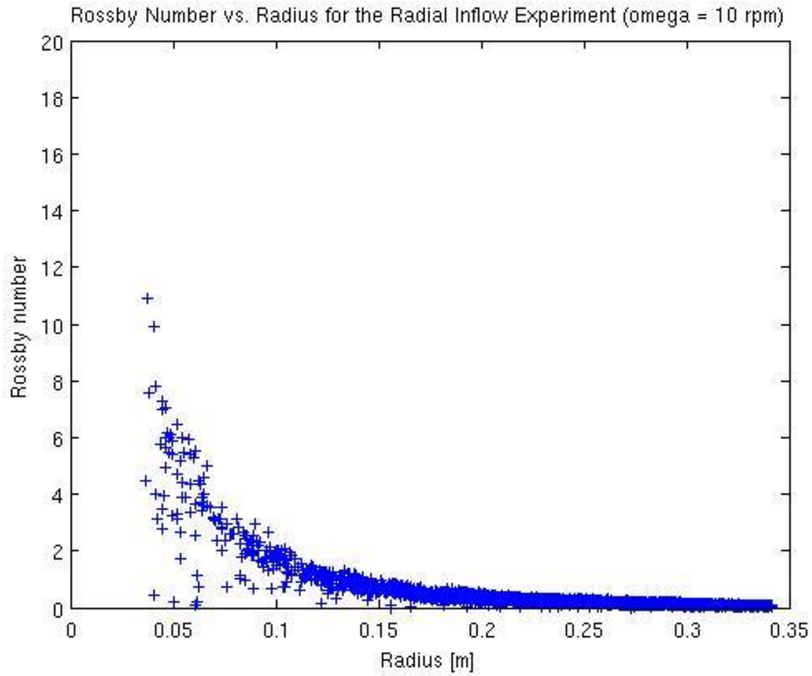


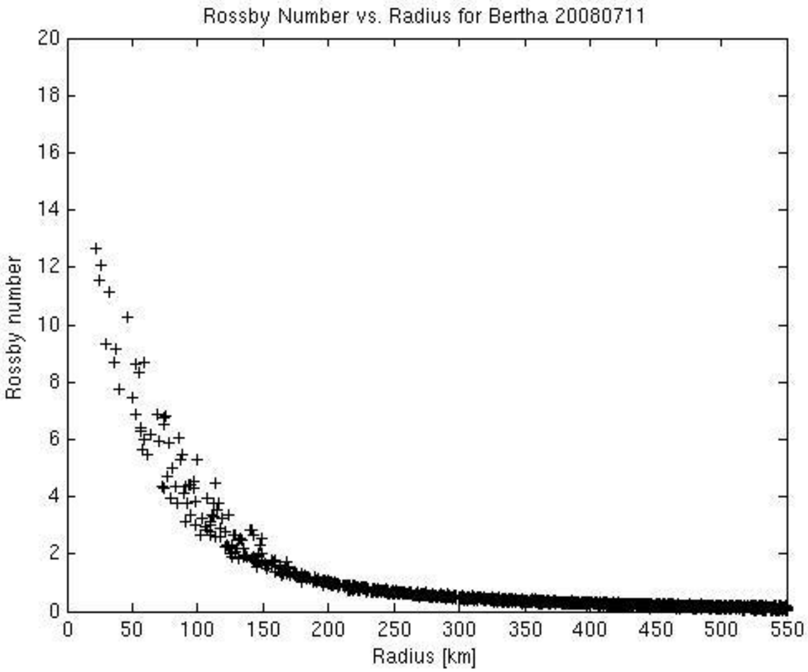
Path of fluid particles in a time dependent flow

- A "recording" of the path of a fluid element in the flow over a certain period.
- The overall path will be determined by the streamlines of the fluid at each moment in time

Do not confuse them!

Balanced Vortex Experiment and the Hurricane flow





How to best combine the two graphs?

Presentations instructions

- Introduction
- Experimental procedure and relevant theory
- Preliminary results
- Atmospheric data
- Connections
- Future work
- Summary

You can include- challenges/next steps/thoughts…