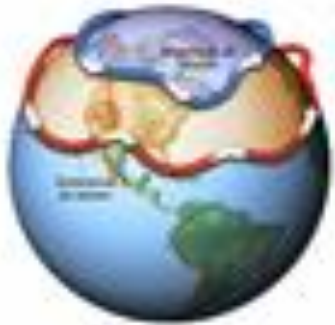
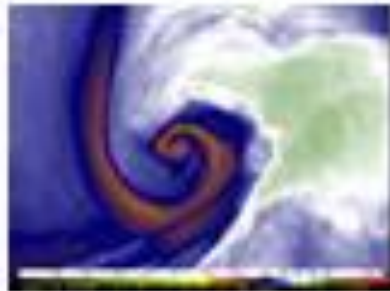


P1: Vortices in the atmosphere

<http://weatherclimatelab.mit.edu/projects/weather-and-extremes/observation-data>



jet stream



blizzard



hurricane



tornado

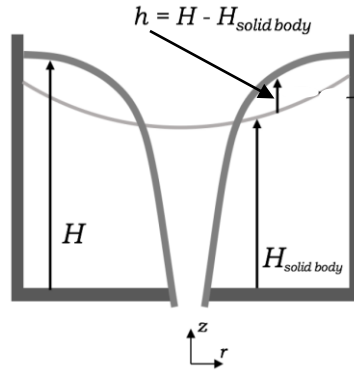
Summary of theory

Inertial Frame

$$\frac{V_\theta^2}{r} = g \frac{\partial H}{\partial r}$$

$$Ro = \frac{|v_\theta^2/r|}{|2\Omega v_\theta|} = \frac{|v_\theta|}{2\Omega r}$$

$$V_\theta = v_\theta + \Omega r$$



Rotating Frame

$$\frac{v_\theta^2}{r} + 2\Omega v_\theta = g \frac{\partial h}{\partial r}$$

$$R_{timescales} = \frac{2\pi/\Omega}{2\pi r/v_\theta} = \frac{v_\theta}{\Omega r} = 2 \times Ro$$

Three limits:

$$Ro \ll 1$$

$$2\Omega v_\theta = g \frac{\partial h}{\partial r}$$

Geostrophic balance

$$Ro \sim 1$$

$$\frac{v_\theta^2}{r} + 2\Omega v_\theta = g \frac{\partial h}{\partial r}$$

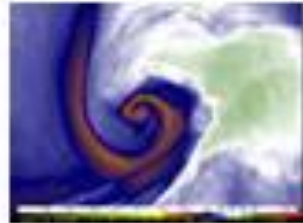
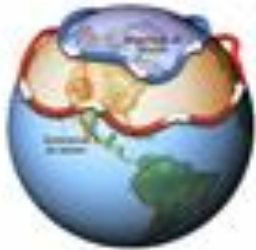
Gradient wind balance

$$Ro \gg 1$$

$$\frac{v_\theta^2}{r} = g \frac{\partial h}{\partial r}$$

Cyclostrophic balance

Atmospheric vortices: balance of forces



$Ro = 0.1$

Rotation Important

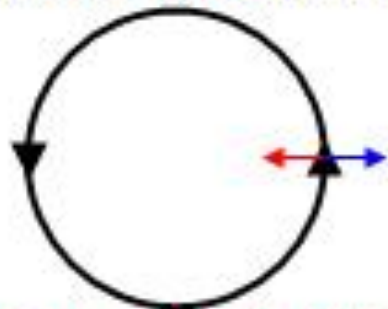
1

Both Important

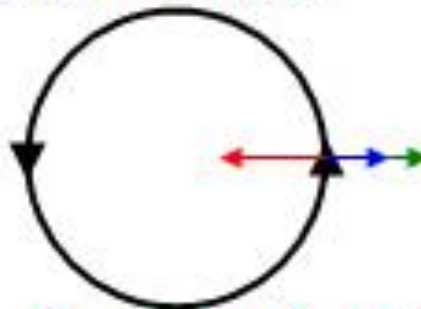
10

Centrifugal Force Important

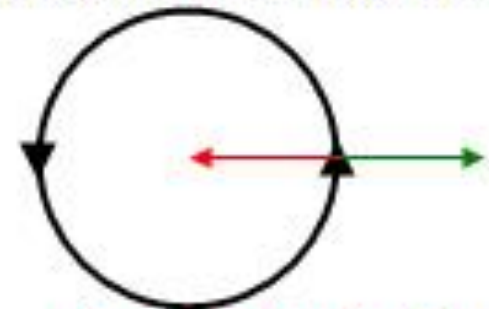
$\Sigma F = 0$



Pressure Gradient Force =
Coriolis Force



Pressure Gradient Force =
Coriolis Force +
Centrifugal Force



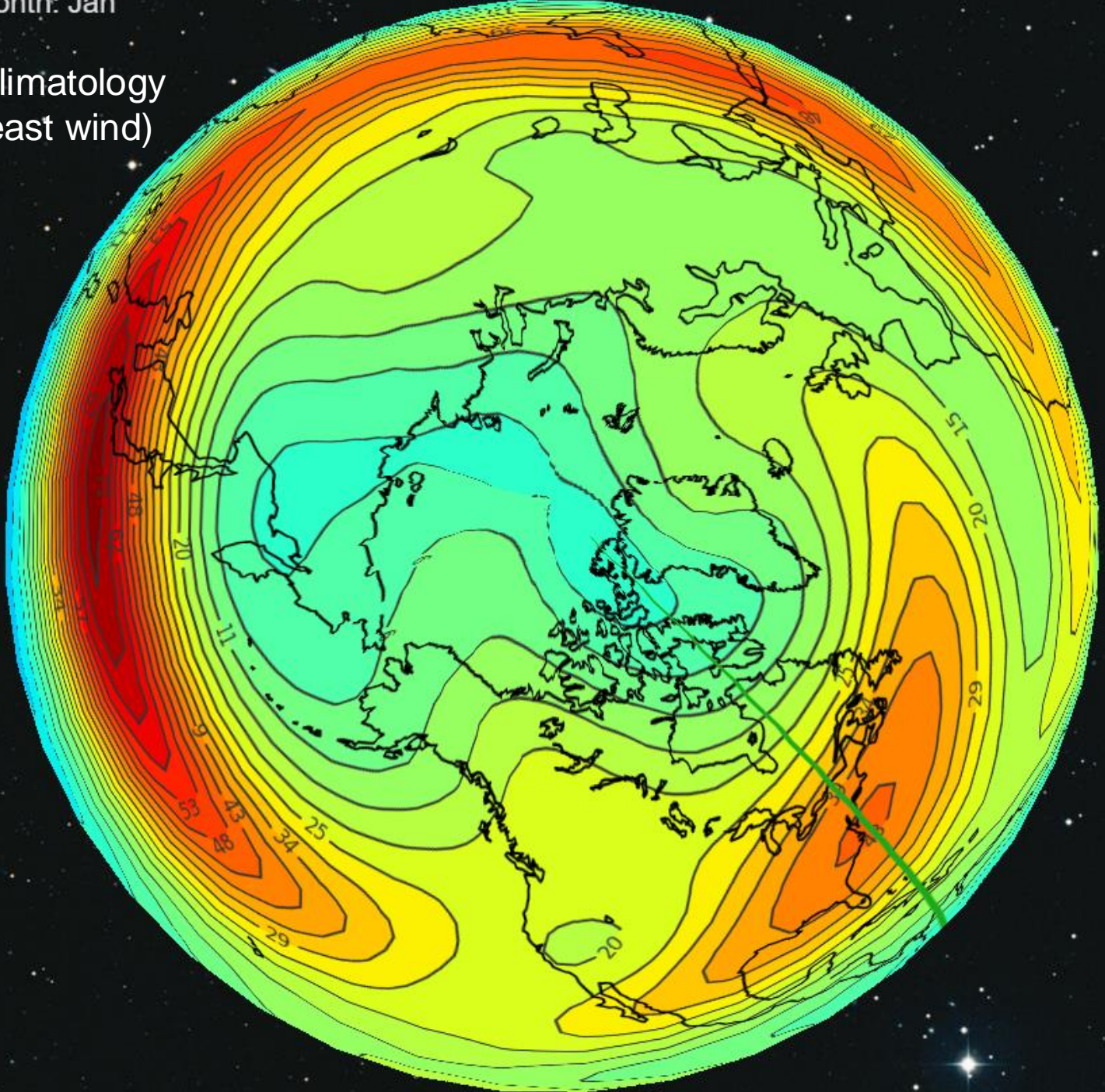
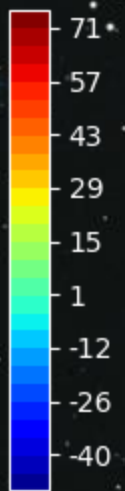
Pressure Gradient Force =
Centrifugal Force

Is the jet stream in geostrophic balance?

Use January climatology to verify it

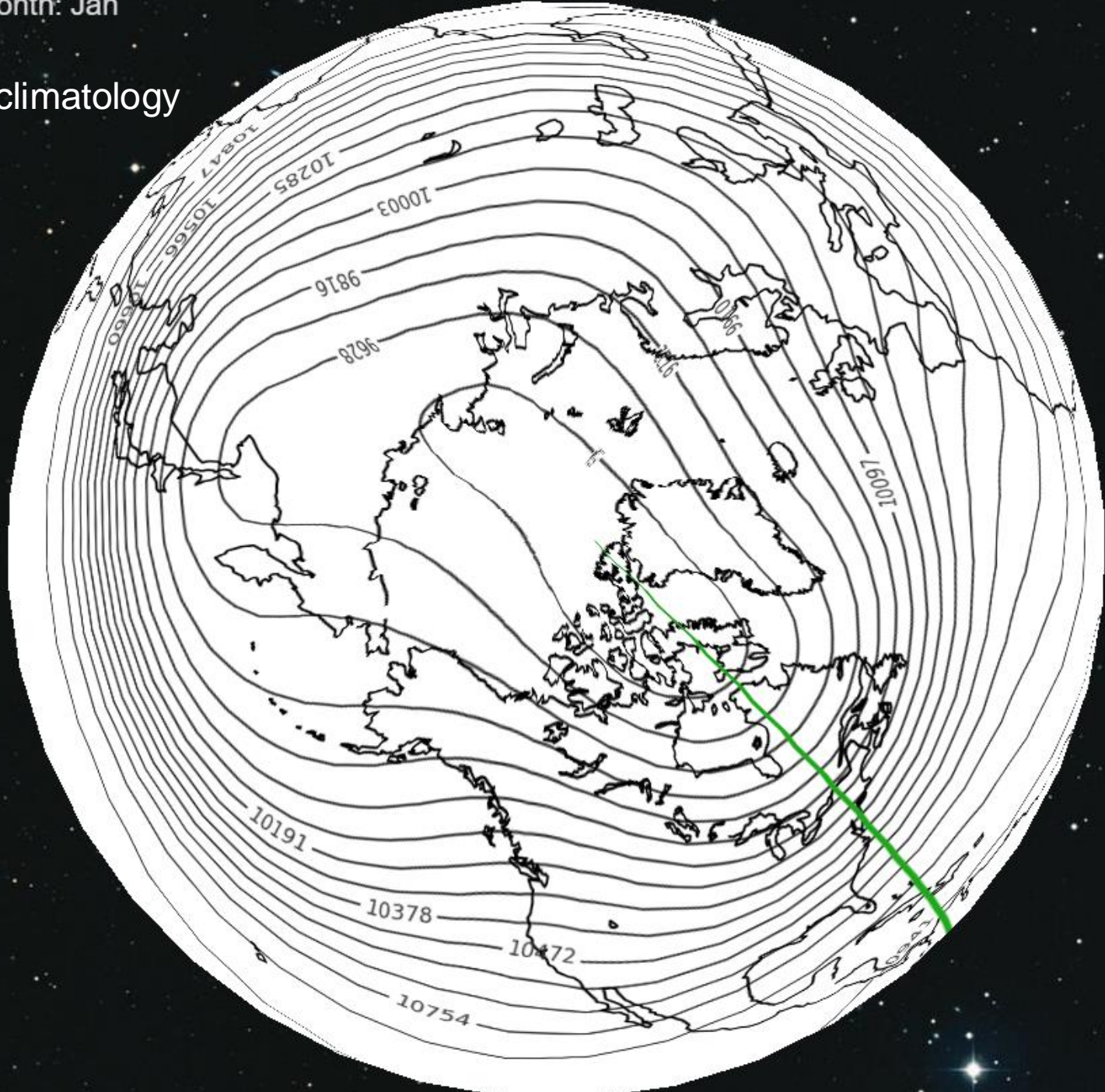
Level: 250, Month: Jan

Uwind - climatology
(west to east wind)



Level: 250, Month: Jan

Height climatology



Jet stream in geostrophic balance ?

$$2\Omega v_{\theta} = g \frac{\partial h}{\partial r}$$

Coriolis Force

Pressure Gradient Force

$$v_{\theta} = \frac{g}{2\Omega} \frac{\partial h}{\partial r} \simeq \frac{g}{2\Omega} \frac{\Delta h}{\Delta r}$$

Put numbers in...

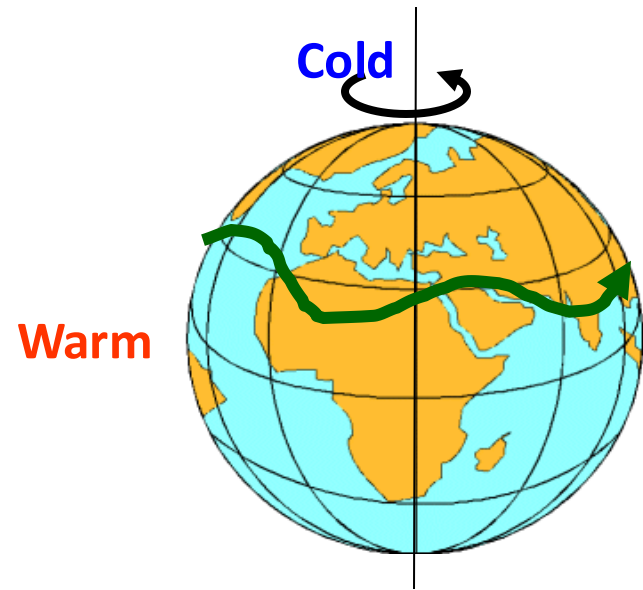
$$\Delta h \cong 1.2 \cdot 10^3 \text{ m}$$

$$\Delta r \cong 3 \cdot 10^3 \text{ km}$$

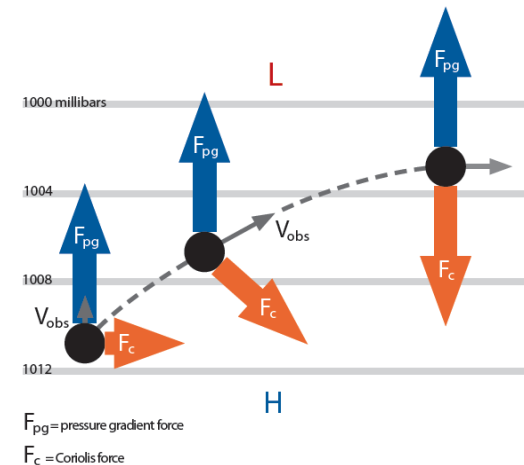
$$v_{\theta} \cong 30 \frac{\text{m}}{\text{sec}}$$

Why is the jet stream generated?

The Equator-to-pole temperature difference induces a meridional (north-south) pressure gradient, with a **Low** pressure over the **Cold** Pole



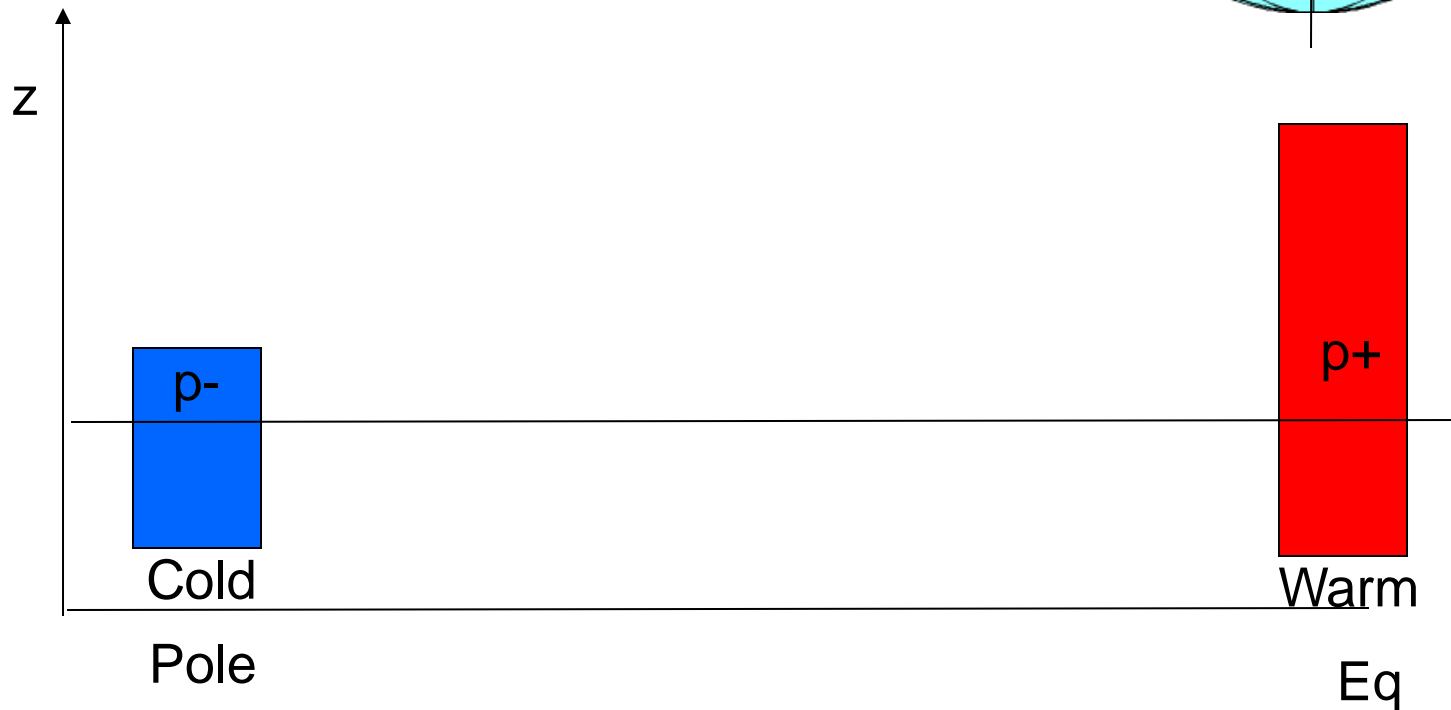
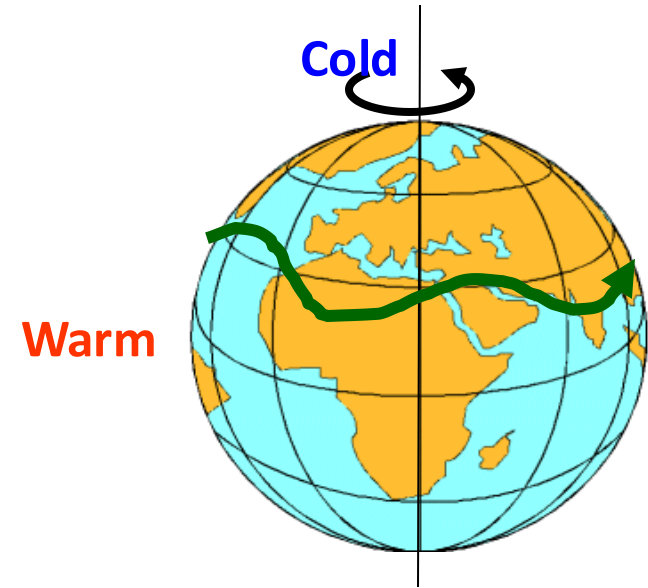
The jet stream is therefore in *geostrophic balance*: the pressure gradient force is balanced by the Coriolis force



At altitude, friction with the Earth lessens and the pressure gradient and the Coriolis forces balance out.

Why is the jet stream strongest at upper levels?

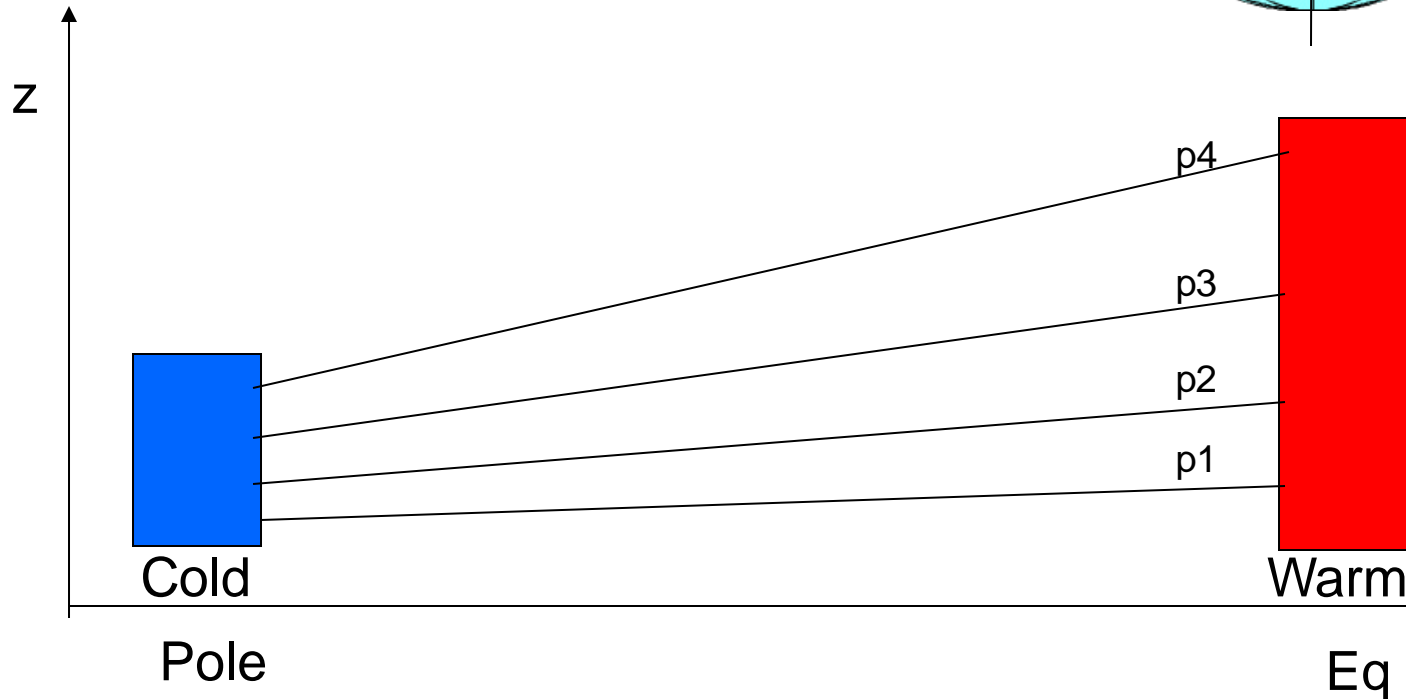
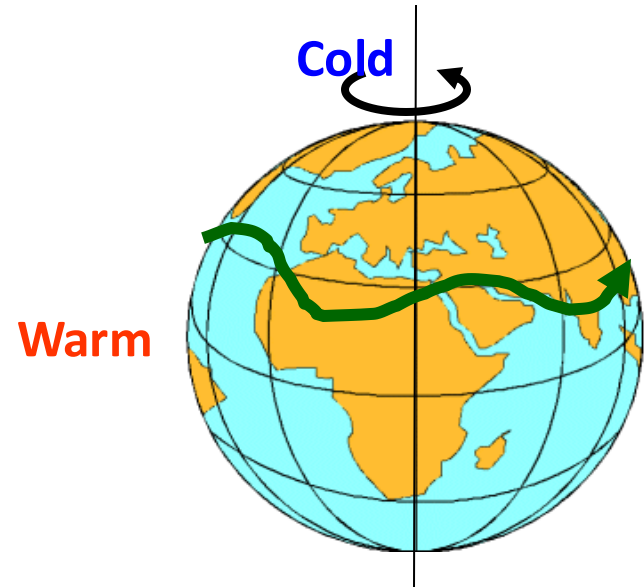
- Because of the N-S temperature difference, pressure gradient force is increasing with height.



Why is the jet stream strongest at upper levels?

- Because of the N-S temperature difference, pressure gradient force is increasing with height.
- The geostrophic wind is therefore also increasing with height (reaching a maximum at the tropopause)

→ *Thermal wind balance!*



Extratropical weather systems

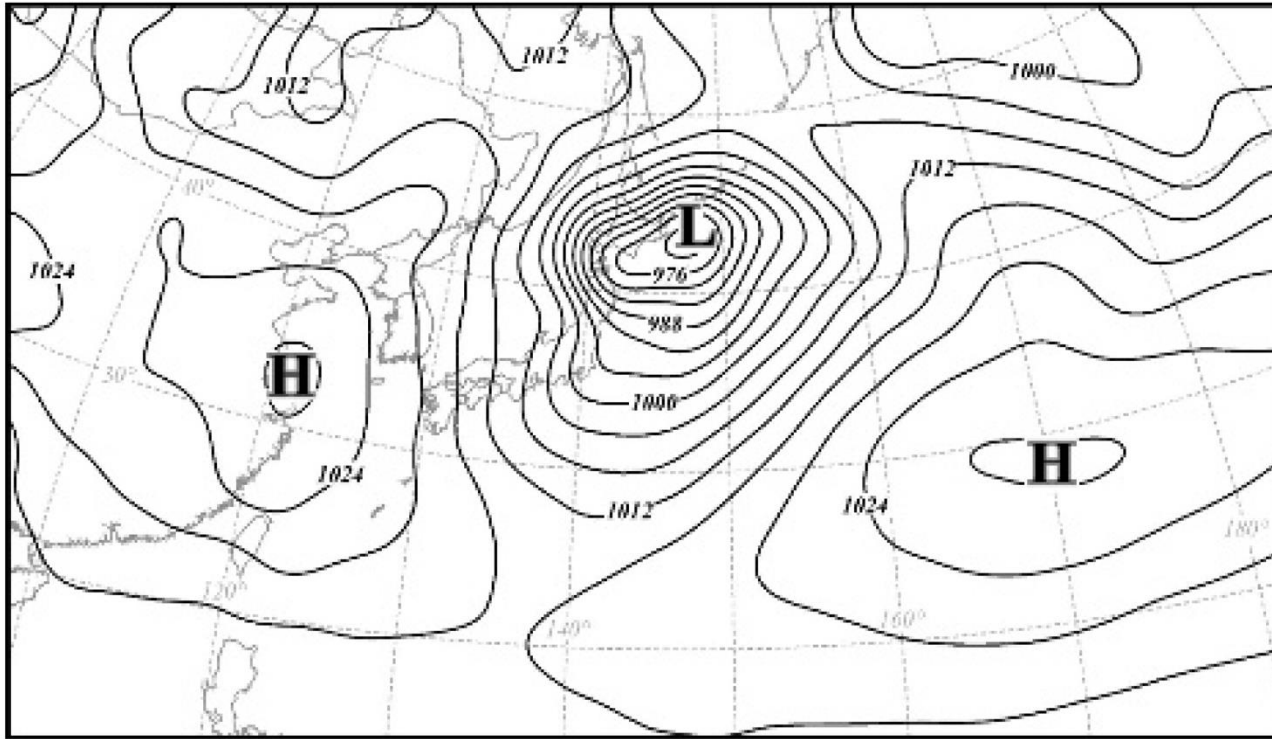


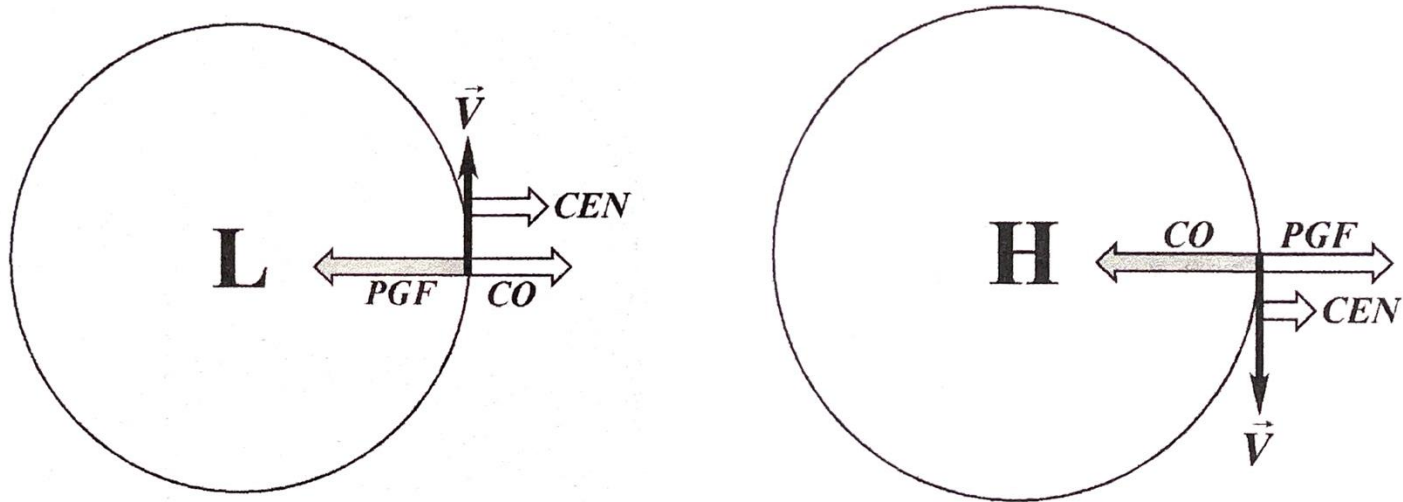
Figure 4.20 Sea-level pressure analysis for 0000 UTC 23 February 2004. Solid lines are isobars labeled in hPa and contoured every 4 hPa. Capital L and H represent centers of sea-level low- and high-pressure systems, respectively. Note the tight pressure gradient around the low and the much weaker pressure gradient around the highs

Figure taken from “Mid-Latitude Atmospheric Dynamics: A First Course”, book by Jonathan E. Martin

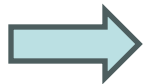
Why are anticyclones (H) weaker and spatially larger than cyclones (L)?

The gradient wind balance-

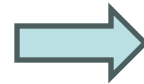
Balance of forces for cyclones (L) and anticyclones (H)



$$\frac{v^2}{r} + fv = g \frac{\partial h}{\partial r}$$



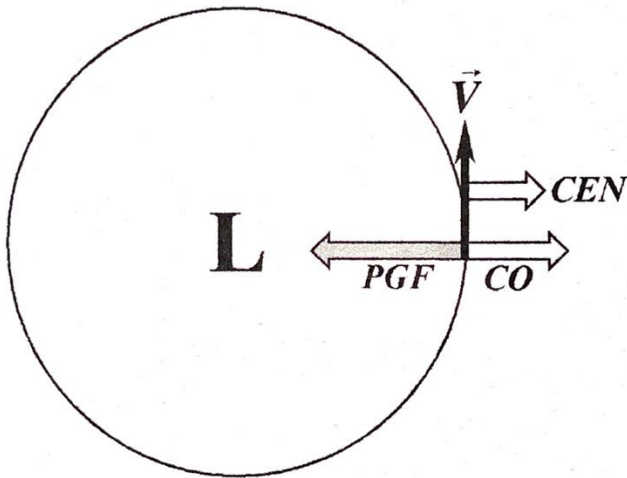
$$\frac{v^2}{r} + fv - g \frac{\partial h}{\partial r} = 0$$



$$\frac{v^2}{r} - f|v| + g \left| \frac{\partial h}{\partial r} \right| = 0$$

The gradient wind balance-

Balance of forces for cyclones (L):



$$\frac{v^2}{r} + fv - g \frac{\partial h}{\partial r} = 0$$

Note that $fv_g = g \frac{\partial h}{\partial r}$

➡ $f(v_g - v) = \frac{v^2}{r} > 0$

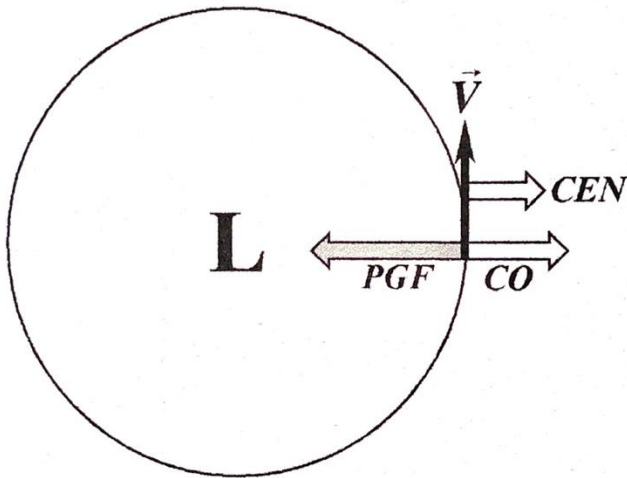
➡ $v < v_g$

Subgeostrophic

The cyclonic wind is always weaker than the geostrophic wind!

The gradient wind balance-

Balance of forces for cyclones (L):



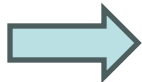
$$\frac{v^2}{r} + fv - g \frac{\partial h}{\partial r} = 0$$



$$v = \frac{-fr}{2} \pm \sqrt{\frac{f^2 r^2}{4} + rg \frac{\partial h}{\partial r}}$$

$$v_1 = -\frac{fr}{2} + \sqrt{\frac{f^2 r^2}{4} + rg \frac{\partial h}{\partial r}} > 0$$

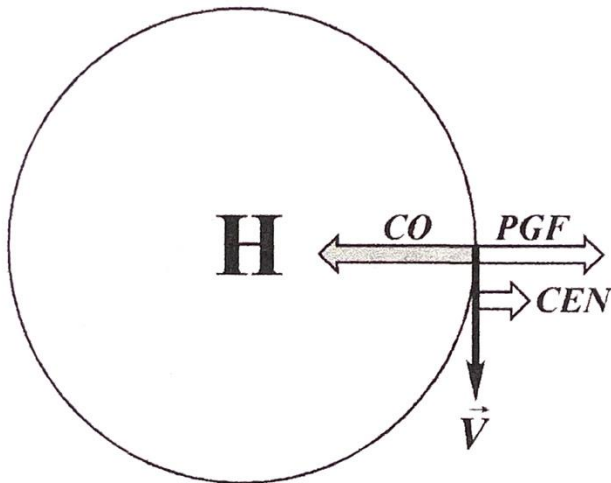
$$v_2 = -\frac{fr}{2} - \sqrt{\frac{f^2 r^2}{4} + rg \frac{\partial h}{\partial r}} < 0$$



$$v = -\frac{fr}{2} + \sqrt{\frac{f^2 r^2}{4} + rg \frac{\partial h}{\partial r}}$$

The gradient wind balance-

Balance of forces for **anticyclones (H)**:



$$\frac{v^2}{r} + fv - g \frac{\partial h}{\partial r} = 0$$

Note that $fv_g = g \frac{\partial h}{\partial r}$

➡ $f(|v| - |v_g|) = \frac{v^2}{r} > 0$

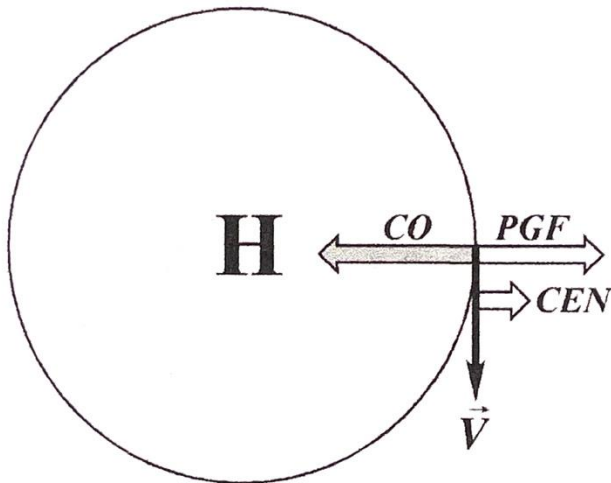
➡ $|v| > |v_g|$

Supergeostrophic

The anticyclonic wind is always stronger than the geostrophic wind!

The gradient wind balance-

Balance of forces for anticyclones (H):



$$\frac{v^2}{r} + fv - g \frac{\partial h}{\partial r} = 0$$

$$v = -\frac{fr}{2} \pm \sqrt{\frac{f^2 r^2}{4} - rg \left| \frac{\partial h}{\partial r} \right|}$$

Solutions only if: $\frac{f^2 r^2}{4} - rg \left| \frac{\partial h}{\partial r} \right| > 0$

$$g \left| \frac{\partial h}{\partial r} \right| < \frac{f^2 r}{4}$$

$$v = \frac{-fr}{2} - \sqrt{\frac{f^2 r^2}{4} - rg \left| \frac{\partial h}{\partial r} \right|}$$

The pressure gradient of an anticyclone is bounded!

Also, $\left| \frac{\partial h}{\partial r} \right| \rightarrow 0$ when $r \rightarrow 0$

Let's check if this simple theory works for real H and L!

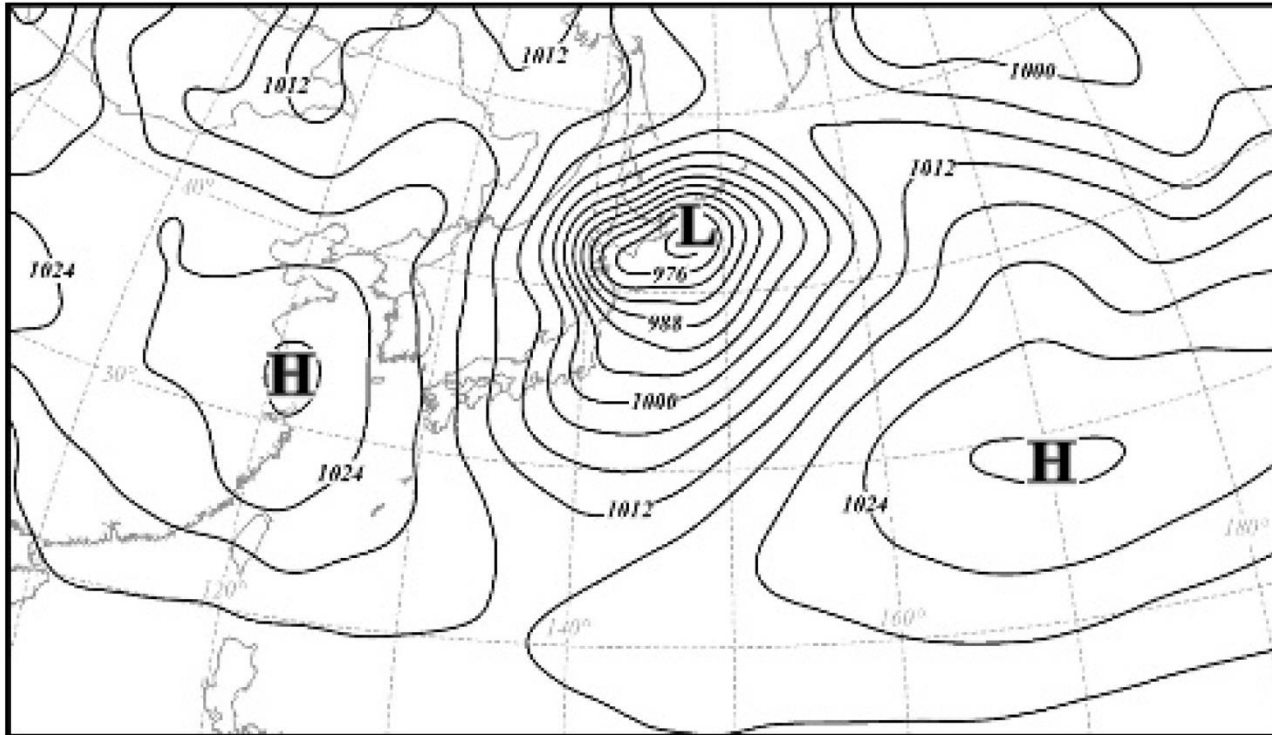


Figure 4.20 Sea-level pressure analysis for 0000 UTC 23 February 2004. Solid lines are isobars labeled in hPa and contoured every 4 hPa. Capital L and H represent centers of sea-level low- and high-pressure systems, respectively. Note the tight pressure gradient around the low and the much weaker pressure gradient around the highs

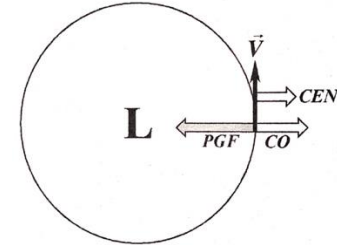
Figure from “Mid-Latitude Atmospheric Dynamics: A First Course”, book by Jonathan E. Martin

Note: the fact that $v < v_g$ for cyclones and $|v| > |v_g|$ for anticyclones **does not** mean that anticyclones velocities are larger. **This is true only for the same pressure gradient!**

Note also that-

For cyclones, we had

$$\frac{v^2}{r} + fv - fv_g = 0$$



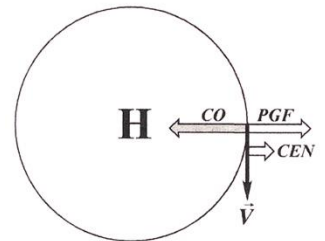
→ $\frac{v^2}{fr} = v_g - v$

→ $\frac{v_g}{v} - 1 = \frac{v}{fr} \equiv R_0$

For cyclones: $\frac{v_g}{v} = R_0 + 1$

→ $v < v_g$

Similarly, for anticyclones we get $\frac{v^2}{r} - f|v| + f|v_g| = 0$



For anticyclones: $\frac{|v_g|}{|v|} = 1 - R_0$

→ $|v| > |v_g|$

Let's check if this simple theory works for real H and L!

We will use the Synoptic Laboratory website (Lodo Illari) to plot v, v_g for real atmospheric data.

- Check February 16th 2024, around Washington (WA state) for a nice example.
 - Go to <http://synoptic.mit.edu/custom-plots/anyscalarwind/>
 - Set:
 1. In the “Scaler” field, change “tmpc” to “hght”
 2. Set the day to “16” instead of today, and year to 2024.
 3. In the “Wind-skip” option, change to yes (to reduce the number of arrows)
 4. In the GAREA option, change “usnps” to “WA--”.
- This will produce a map with wind barbs and the 500mb geopotential height
- Now repeat, but in the “Wind” option change “observed” to “Geostrophic”

- Is the actual v smaller or larger than the geostrophic velocity in the L/H region?

- Can you estimate the Rossby number?

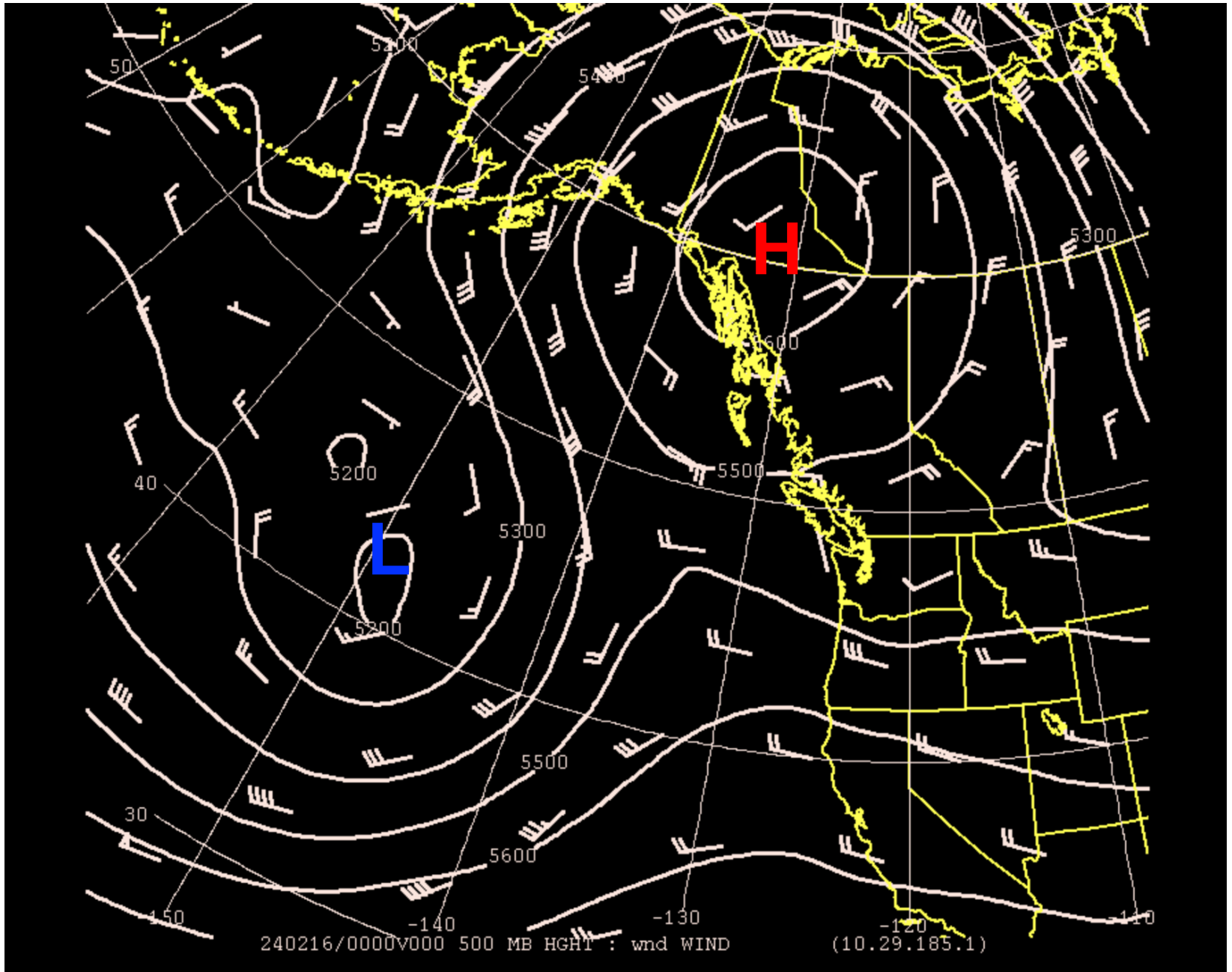
cyclones

$$R_0 = \frac{v_g}{v} - 1$$

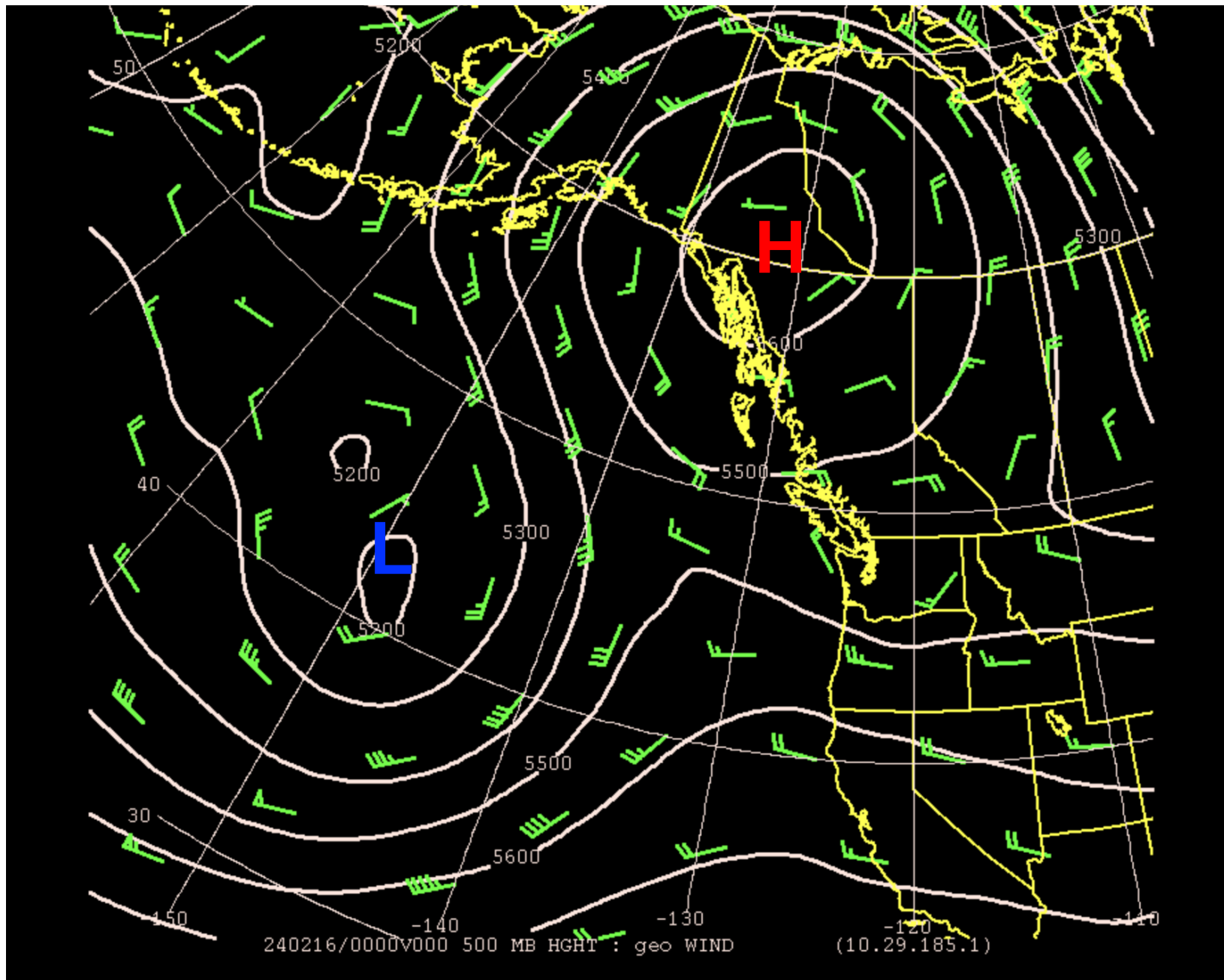
anticyclones

$$R_0 = 1 - \frac{v_g}{v}$$

Total wind



Geostrophic wind



50 + 10 + 10 + 5



Wind blowing from the west at 75 knots



Wind blowing from the northeast at 25 knots



Wind blowing from the south at 5 knots



Calm winds

Hurricanes-

Next class!

EsGlobe uses a **global** dataset:

Winds from the **GFS - Global Forecast Model** (NCEP)

lat, lon grid with a resolution of $\frac{1}{4}$ of degree = ~ 25km

Not enough resolution to represent well an hurricane, which has a radius of few hundreds km

To study the balance of forces in a hurricane we are using a special dataset: surface wind data from the “**scatterometer**” instrument

See [scatterometer instructions](#)

Hurricane flow and the balanced vortex experiment

