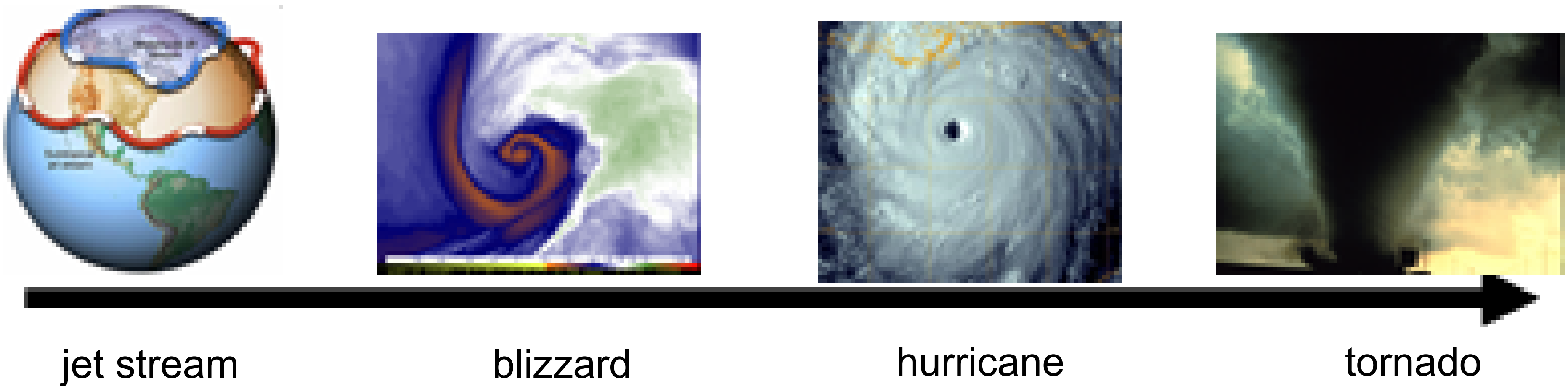
P1: Vortices in the atmosphere

http://weatherclimatelab.mit.edu/projects/weather-and-extremes/observation-data

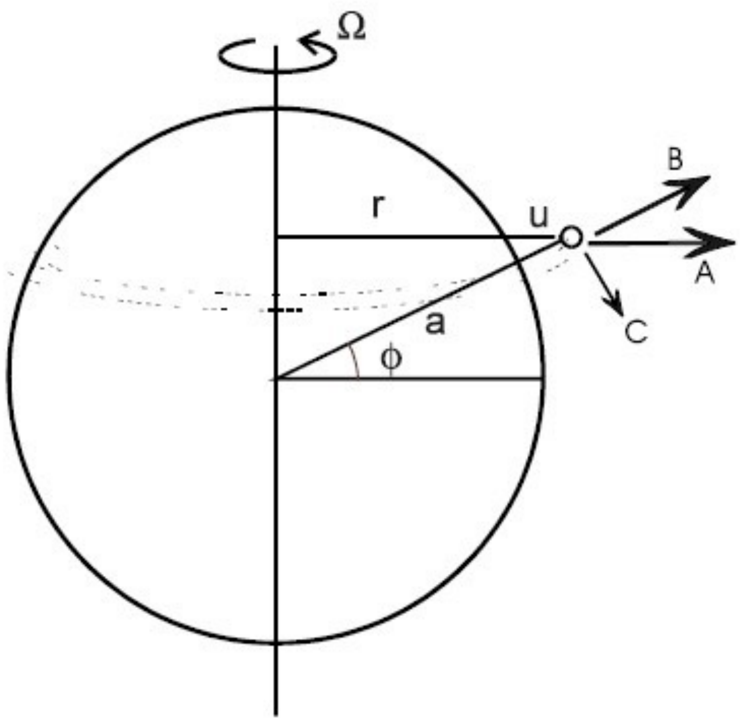


Today:

- Jane Abbott (CIM presentations)- on Thursday at 4pm
- Goestrophic balance on a sphere
- Measurement errors
- Hurricane data (scatterometer)
- Guidelines for presentation

Visualization of the **Coriolis Force**

Let’s consider a ring of air moving from west to east with velocity U



Centrifugal acceleration:

$$A = \frac{V^2}{r} = \frac{(U+\Omega r)^2}{r}$$

$$= \underbrace{\Omega^2 r}_{\text{included in g}} + 2\Omega U + \underbrace{\frac{U^2}{r}}_{\text{small}}$$

To a good approximation

$$A = 2\Omega U$$

‘A’ can be resolved into two components: B and C

B’ is ⊥ to the earth surface
’B’ changes the weight of the ring slightly

‘C’ is ∥ to the earth surface $= 2\Omega \sin\varphi U$

$$\boxed{C = 2\Omega \sin\varphi U}$$

C is the Coriolis acceleration and it’s pointing southward

Visualization of the Coriolis Force

Let's postulate a balance between Coriolis force and the pressure gradient force:

$$\underbrace{2\Omega \sin\varphi U}_{\text{acceleration}} \times \underbrace{\varrho a d\varphi dz}_{\text{mass}} = -\frac{\partial p}{\partial \varphi} d\varphi dz$$

$$dy = a d\varphi$$

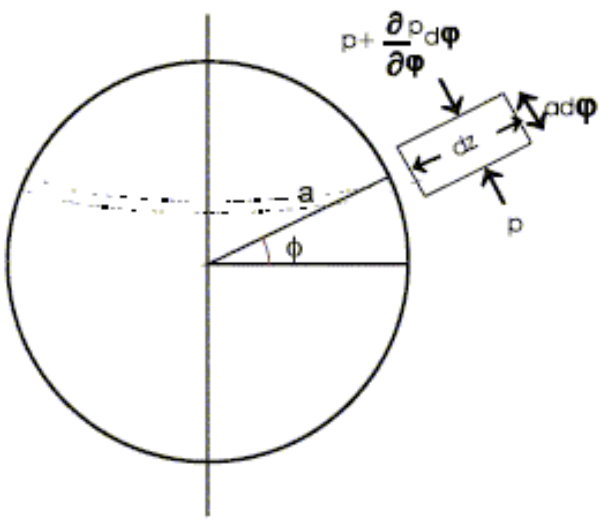


Figure 6:

$$2\Omega \sin\varphi U \times \varrho dy dz = -\frac{\partial p}{\partial y} dy dz$$

$$2\Omega \sin\varphi U = -\frac{1}{\varrho}\frac{\partial p}{\partial y}$$

$$f = 2\Omega \sin\varphi$$

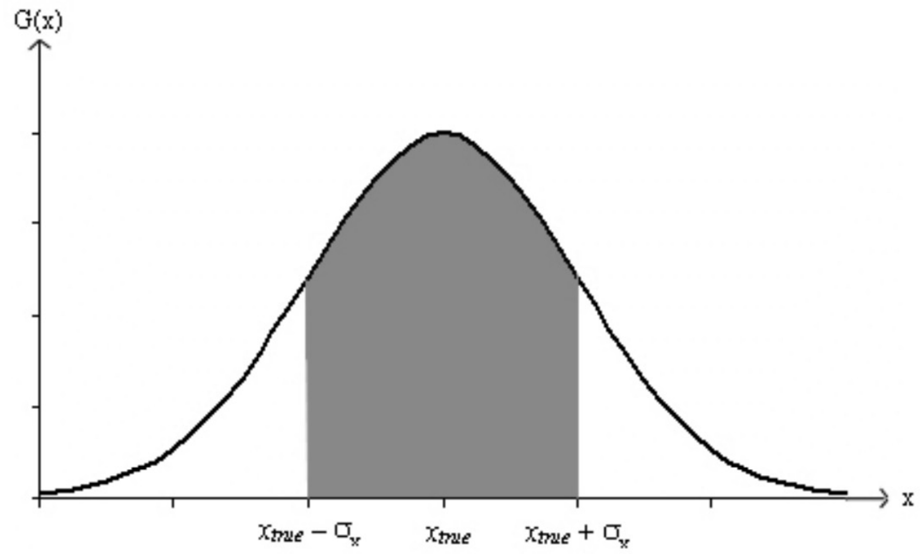
is the Coriolis parameter

This is the geostrophic wind resulting from the balance between the PGF and the Coriolis force

$$U = -\frac{1}{\varrho f}\frac{\partial p}{\partial y}$$

$$V = \frac{1}{\varrho f}\frac{\partial p}{\partial x}$$

Statistical Analysis of Random Uncertainties



$$\bar{x} = \frac{\sum_{i=1}^{N} x_i}{N}$$

Error of a signle measurment-

$$\sigma_x = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{x})^2}$$

For multiple measurments, the error of the mean can be estimated by-

$$\sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{N}}$$

Propagation of Errors

$$y = f(x_1, x_2, \ldots) \Longrightarrow dy = \frac{\partial y}{\partial x_1} dx_1 + \frac{\partial y}{\partial x_2} dx_2 + \ldots$$

$$\Delta y = \sqrt{(dy)^2} = \sqrt{\left(\frac{\partial y}{\partial x_1} dx_1\right)^2 + \left(\frac{\partial y}{\partial x_2} dx_2\right)^2 + \ldots + 2\left(\frac{\partial y}{\partial x_i}\frac{\partial y}{\partial x_j}\right) dx_i dx_j + \ldots}$$

$$\Downarrow$$

$$\boxed{\Delta y = \sqrt{\sum_i \left(\frac{\partial y}{\partial x_i} \Delta x_i\right)^2} = \sqrt{\left(\frac{\partial f}{\partial x_1} \Delta x_1\right)^2 + \left(\frac{\partial f}{\partial x_2} \Delta x_2\right)^2 + \ldots}}$$

Propagation of Errors

For the sum $Q = ax + by$ or difference $Q = ax - by$ (where a, b are constants) of quantities:

$$\Delta Q = \sqrt{(a\Delta x)^2 + (b\Delta y)^2}$$

For the product $(Q = ax \cdot y)$ or division $(Q = a\frac{x}{y})$ (where a is a constant) of quantities:

$$\frac{\Delta Q}{Q} = \sqrt{\left(\frac{\Delta x}{x}\right)^2 + \left(\frac{\Delta y}{y}\right)^2}$$

Hurricanes

EsGlobe uses a **global** dataset:

Winds from the **GFS - Global Forecast Model** (NCEP)

lat, lon grid with a resolution of ¼ of degree = ~ 25km

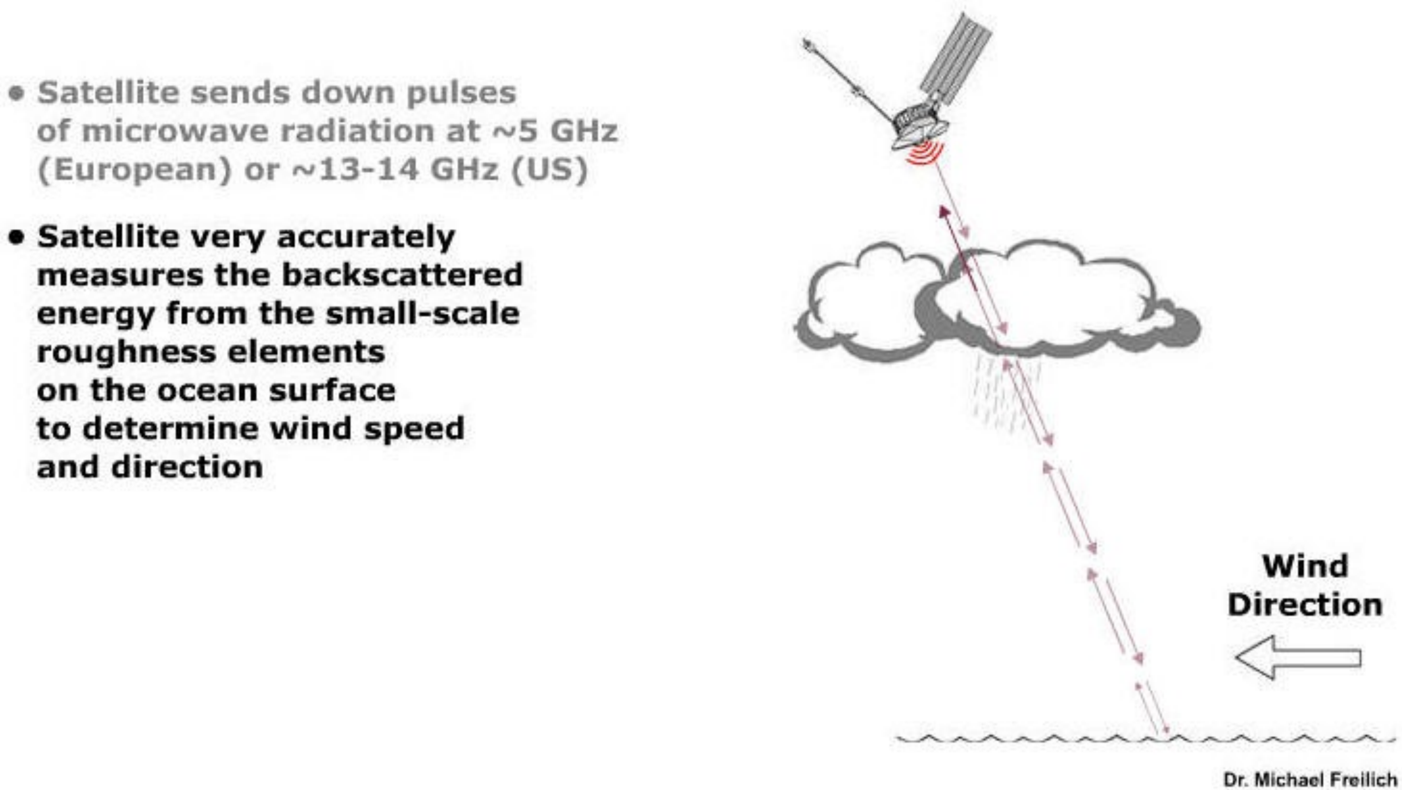
Not enough resolution to represent well an hurricane, which has a radius of few hundreds km

To study the balance of forces in a hurricane we are using a special dataset: surface wind data from the "**scatterometer**" instrument

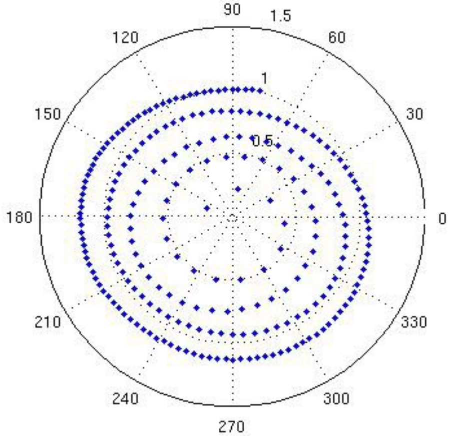
See scatterometer_instructions

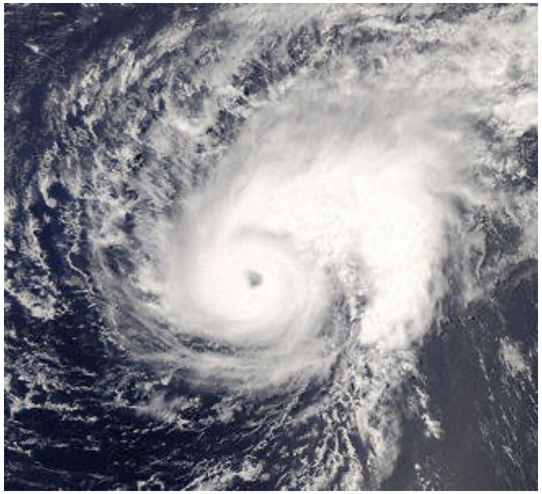
Scatterometer data to analyze Hurricanes!

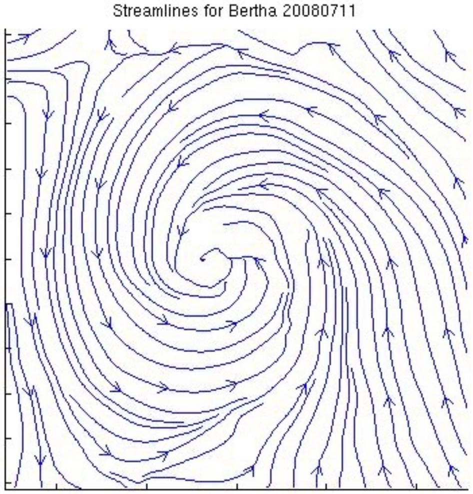
Scatterometers are remote sensing instruments for deriving wind direction and speed from the roughness of the sea. They are used by low Earth orbiting satellites and act like radars: they transmit electromagnetic pulses and detect the backscattered signals.



Hurricane flow and the balanced vortex experiment



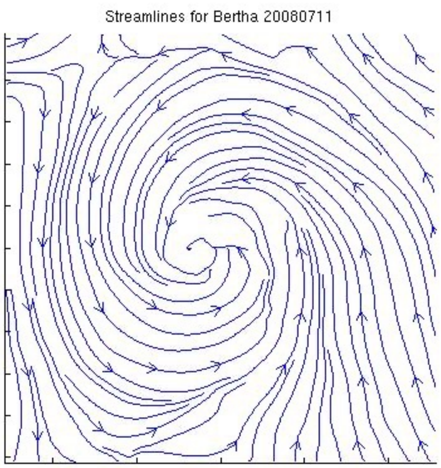




Streamlines:

Lines which smoothly connect the velocity vectors at an instance in time

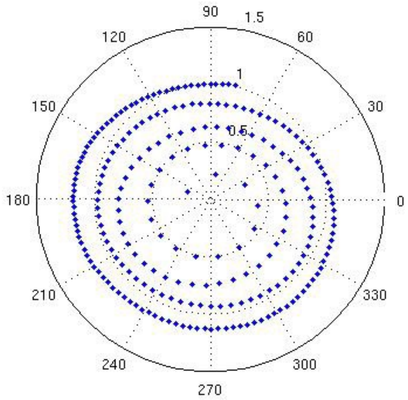
- Shows the direction in which a fluid element will travel in an instantaneous flow
- Can change with time (unless the flow is steady)



Trajectories:

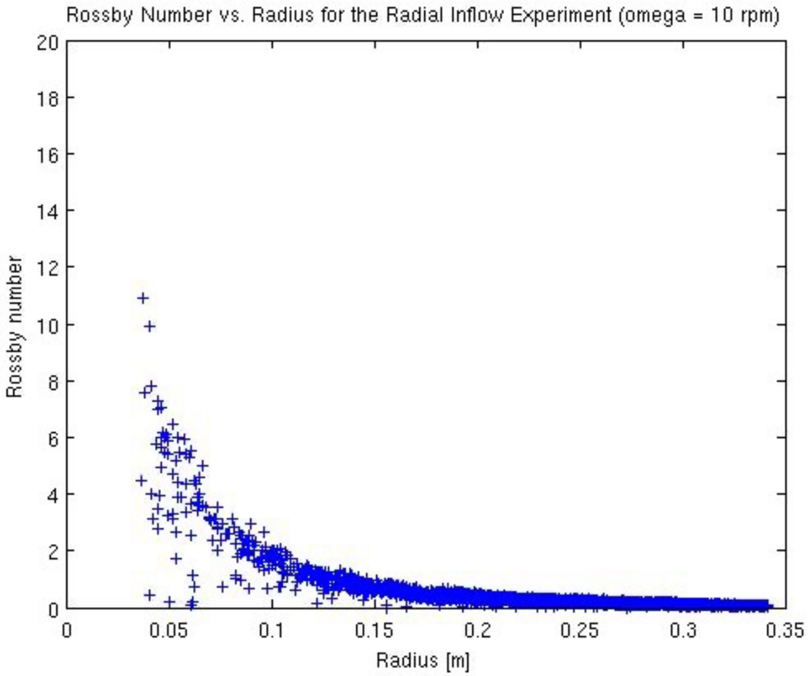
Path of fluid particles in a time dependent flow

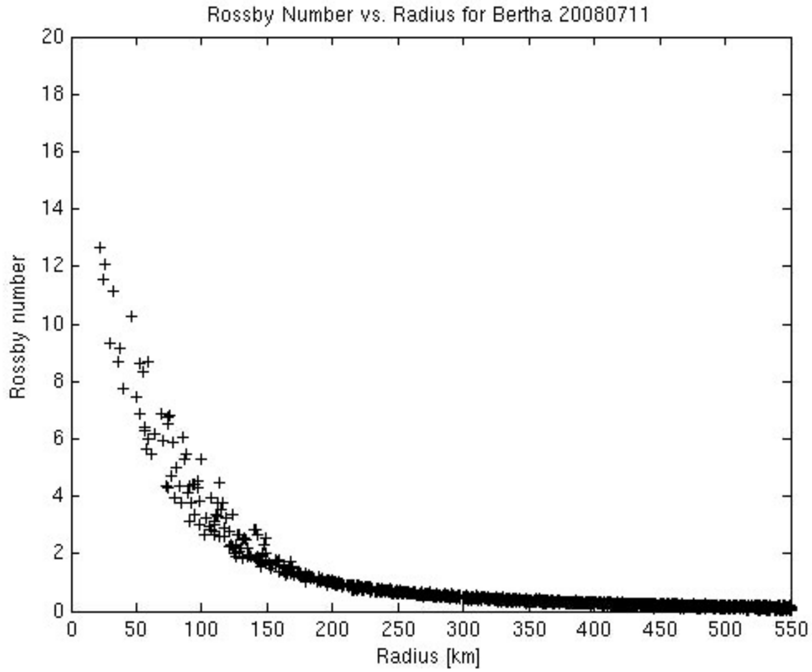
- A "recording" of the path of a fluid element in the flow over a certain period.
- The overall path will be determined by the streamlines of the fluid at each moment in time



Do not confuse them!

Balanced Vortex Experiment and the Hurricane flow





How to best combine the two graphs?