3 Vortices in the atmosphere

We will make use of what we have learned about the nature of laboratory vortices in rotating systems to explore atmospheric vortices using atmospheric data. Just as in our study of laboratory vortices, the Rossby number will be the key non-dimensional number that threads through our exploration. We begin by estimating the Rossby number in atmospheric vortices on large and small scales.

3.1 Examples of intense atmospheric vortices on large and small scales

The atmosphere is full of swirling wind patterns which often comprise intense vortex structures, as can be seen in Fig.10, a global IR satellite image of clouds on Feb 11, 2019. We observe vortical patterns of varying sizes, from large ones with scales of thousands of kilometers to small-scale vortices within which are embedded intense convective clouds clusters and thunderstorms. Tornadoes, which have a scale of only few km, can often develop in regions of severe thunderstorms, but are too small to be seen in such global images.

Many types of atmospheric vortices exist, which can be organized as a function of scale, from large to small, as listed below:

The Jet Stream (scale = 3000 km): a current of fast-moving winds at the height of 10 km or so, flowing from west to east around a low pressure area located over the cold polar regions. It is sometimes called the "Polar Vortex" — see Fig.11 and accompanying video. A similar jet stream flows around the South Pole.

Blizzards (scale = 1000 km) — winter snow storms associated with strong winds circulating anti-clockwise around a low pressure center. They tend to develop along the "Polar Front", a mid-latitude region of strong temperature gradient, marking the boundary between cold polar air and warm tropical air — see Fig.12.

Hurricanes (scale = few 100s km): — tropical storms associated with very strong winds circulating anti-clockwise around a very low pressure center, the 'eye' of the hurricane. Hurricanes are fueled by energy in warm surface waters of the ocean and tend to occur in the summer/fall season — see Fig.13.

Tornados (scale = 1 km) — small-scale vortices with extremely strong, damaging winds, which can develop out of strong thunderstorms — see Fig.14. In a tornado the air can swirl either clockwise or anti-clockwise.

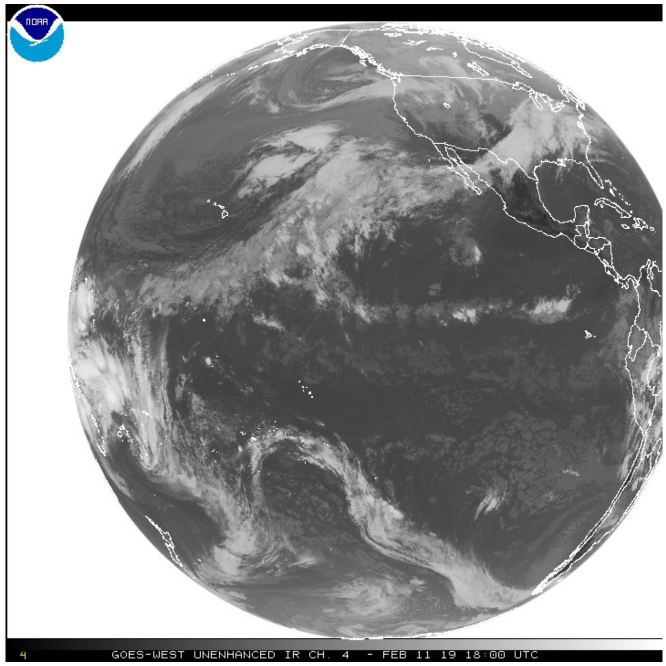

Figure 10: A global satellite image of Earth from Feb 11, 2019. The white areas are clouds and the darker areas clear sky. You can view an accompanying movie loop here: https://www.goes.noaa.gov/dml/west/fd.

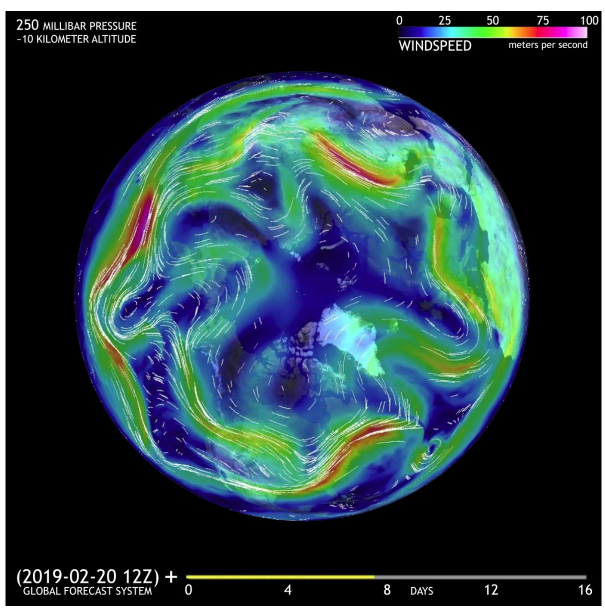

Figure 11: The jet stream at 250 HPa (at a height of, roughly, 10 km) as seen on Feb 20, 2019. The areas shaded in red correspond to wind speeds that exeed $50\,\mathrm{m\,s^{-1}}$. The jet stream is in constant motion, as the 'ribbon' of intense winds undulates north and south, as can be seen in the accompanying movie.

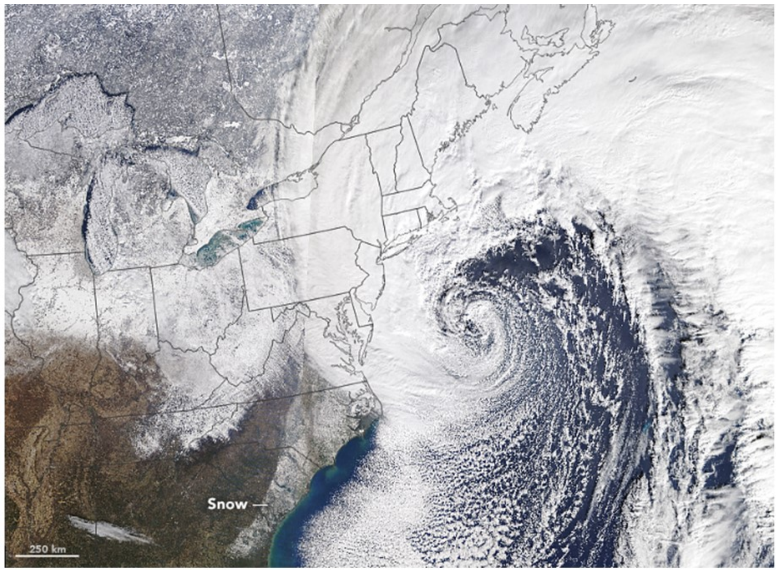

Figure 12: A satellite image of clouds showing the system that brought the blizzard of January 4, 2018 to the east coast. We see cold air moving down from the Arctic on the west of the system and warm air moving up from the tropics on the east — for more see: https://www.weather.gov/okx/Blizzard_Jan42018.

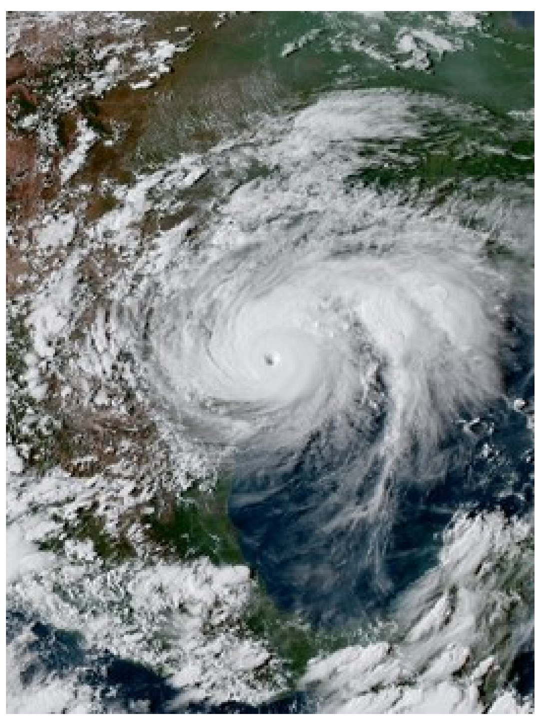

Figure 13: Hurricane Harvey (category 5) which reached landfall on the coast of Texas on August 25, 2017 — for more see: https://earthobservatory.nasa.gov/images/90822/hurricane-harvey-approaches-texas

Figure 14: A tornado in Iowa on July 19, 2018, one of 19 tornadoes that developed over the central planes. For more see: https://www.weather.gov/dmx/20180719_Tornadoes.

3.2 Estimating the Rossby number as a function of scale

We will now estimate the Rossby number of various atmospheric vortices, in a manner which is exactly analogous to that used in our laboratory experiments in Section 1.1. Rather than using paper dots, we will launch 'virtual' particles in to the evolving wind patterns using the EsGlobe, an interface for displaying and interacting with data — 12.307 website.

Planetary scale – the jet stream. Use the EsGlobe particle tracking interface to explore how long it takes for a parcel of air in the jet stream to circumnavigate the globe (set level = 10 km $\sim$ 250 mb, where the jet speed typically reaches a maximum). Estimate the $R_{timescales}$ (as a ratio of time scales) defined by Eq.(1). Note that the Rossby number, Eq.(12), is given by $R_o = \frac{1}{2} R_{timescales}$ for an axi-symmetric vortex (see footnote on page 11). Is the Rossby number large or small and what does that tell us about the balance of forces that maintains the jet stream?

Mid-latitude cyclones — blizzards and Nor'easters. How long does it take for a parcel of air associated with a blizzard (or a mid-latitude cyclone) to travel full circle around the low pressure center? Use the same trajectory interface but set level = 5 km, $\sim$ 500 mb, the level that sets the speed of propagation of mid-latitude cyclones. Again, estimate the Rossby number and discuss the typical balance of forces in a large-scale blizzard.

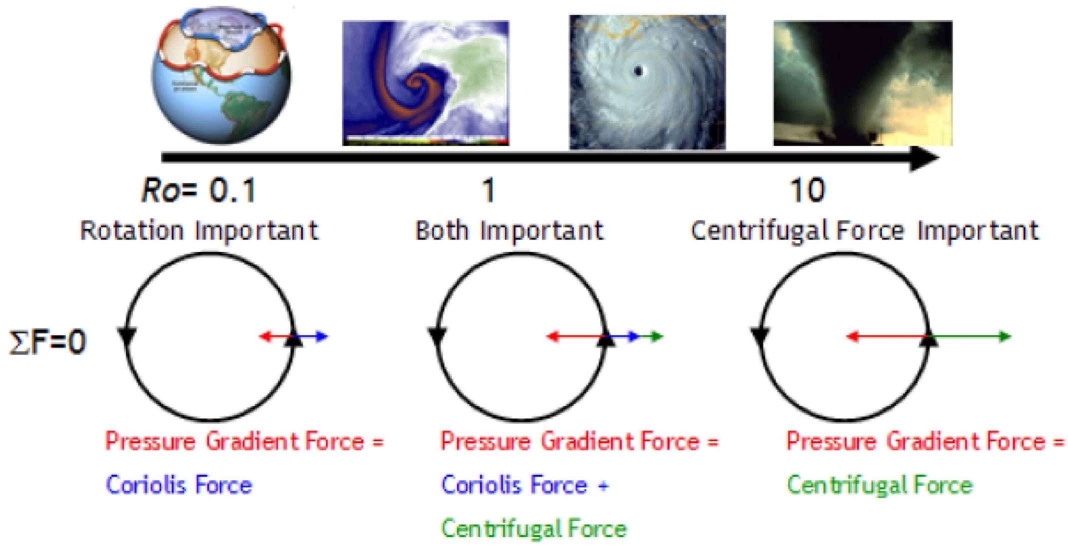

Figure 15: Significance of Rossby number as it relates to the balance of forces in vortices of different scales. On the planetary scale (lhs) $R_o << 1$ and pressure gradient forces are balanced by Coriolis forces. On the scale of a tornado (rhs) $R_o >> 1$ and pressure gradient forces are balanced by centrifugal forces.

Tropical cyclones — hurricanes. Repeat the same procedure but for hurricane Harvey (August, 2018). Use the Harvey winds available from the EsGlobe interface and set level = 1 km , $\sim$ 850 mb, the level at which hurricane winds are the strongest. Estimate the Rossby number and discuss the typical balance of forces for a hurricane.

What about a tornado? Discuss the likely balance of forces.

In summary you should find that R_o depends on scale, broadly as sketched in Fig.15. On the large-scale, $R_o << 1$, and the dynamics is dominated by Earth's rotation, i.e. the Coriolis force. In this case the dominant radial force balance is given by Eq.(13). In contrast, if $R_o >> 1$, then centrifugal forces acting on the fluid parcel are dominant. In this case the dominant radial force balance is given by Eq.(15). In each of these cases, the force directed radially-outwards from the center of the vortex (whether Coriolis or centrifugal) is balanced by the pressure gradient force directed inwards, toward the low pressure at the center.

3.3 Jet stream in Geostrophic balance

Use the EsGlobe Atmospheric climatology interface to plot the zonal component of the wind field at 250mb, together with the height of the 250mb surface. An example is shown in Fig.16 from a January climatology. The globe has been orientated so that the north pole (NP) is at the center of the plot. Note:

1. the wind blows from west to east, circling cyclonically (anti-clockwise, looking down from the NP) around the pole. The reddest areas are the fastest, reaching speeds of $50\,\mathrm{m\,s^{-1}}$ or so.

2. the 250mb surface is low over the pole (a height of 9600 m or so) and gently rises moving in to the tropics (to a height of 10800 m or so).

3. since $R_o << 1$, the zonal flow is in geostrophic balance with the poleward-directed pressure gradient force and Eq.(13) pertains. Thus, rearranging,

$$v_\theta = \frac{g}{2\Omega}\frac{\partial h}{\partial r} \simeq \frac{g}{2\Omega}\frac{\Delta h}{\Delta r},$$

   where Δh is the increase in the height of the 250mb surface moving outward from the pole, typically 1.5km and Δr is the lateral scale over which it occurs, typically 3000km or so. Plugging in numbers we obtain, including Earth's rotation rate and the acceleration due to gravity:

$$v_\theta = \frac{9.81\,\mathrm{m\,s^{-2}}}{2\times 7\times 10^{-5}\,\mathrm{s^{-1}}}\frac{(10.8-9.6)\times 10^3\,\mathrm{m}}{3\times 10^3\,\mathrm{km}} \simeq 30\,\mathrm{m\,s^{-1}},$$

   or so, of the same order as the observed zonal wind speed — see Fig.16(top).

Connections to our radial inflow experiment should be clear. The center of our bucket represents the NP, a region of low pressure, the tilt of the free surface of the water is analogous to the tilt of atmospheric pressure surfaces, and the anti-clockwise trajectories of the paper dots are analogous and the west to east winds of the jet stream.

3.4 Hurricane flow — balance of forces

We now consider hurricane flow in more detail. A comprehensive list of interesting hurricane cases is available here: — see scatterometer archive of historical storms: http://www.remss.com/storm-watch/#storm-archive.

Choose a hurricane and plot the observed winds, following the scatterometer instructions here: http://weatherclimatelab.mit.edu/scatterometer-instructions. From wind field compute the Rossby number defined as in Eq.(12) $R_o = \frac{|v_\theta|}{fr}$,where $f = 2\Omega\,\mathrm{sinlat}$ is the Coriolis parameter, v_θ is the azimuthal velocity and r is the distance from the center of the hurricane — see Fig. 7.

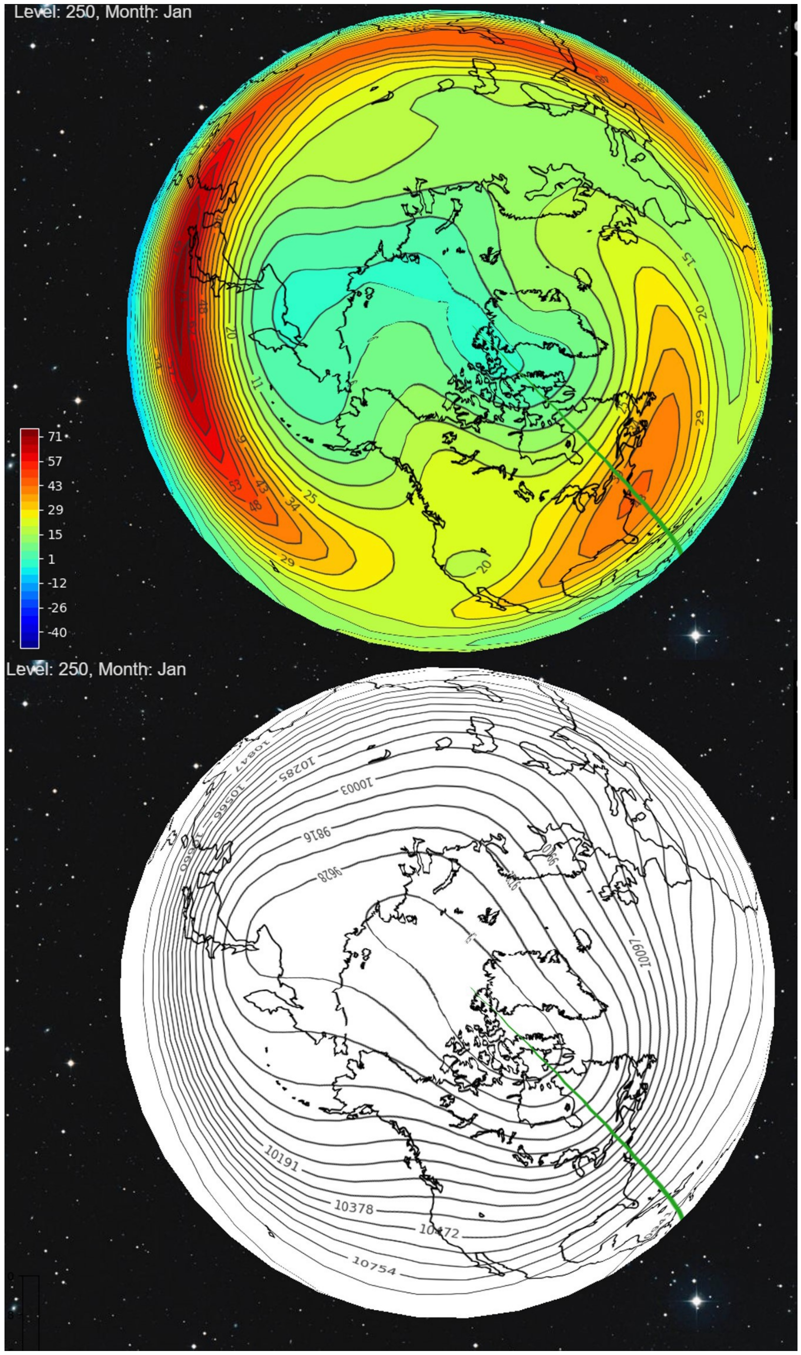

Figure 16: (top) The monthly-averaged January zonal (west to east) wind at 250mb, showing the jet stream. The north pole is in the center. Color scale on the left in $\mathrm{m\,s^{-1}}$. Green and red colors indicate eastward flow. (bottom) The monthly-averaged January height of the 250mb surface in meters varying from a height of 9600m over the pole to10800m over the equator.

3.5 Extratropical cyclones and anticyclones and the gradient wind balance

As we saw, the Rossby number associated with extratropical storm is of the order one, which means that all three forces (pressure gradient, Coriolis, and centrifugal forces) are important.

We next explore in more detail how the force balance for anti-clockwise (cyclonic) and clockwise (anticyclonic) circulations (in the Northern Hemisphere, NH) can help us explain some interesting and fundamental differences between observed cyclones and anticyclones. Fig.1 below shows an example snapshot of observed sea level pressure, with 'L' and 'H' marking the centers of cyclones and anticyclones, respectively. As can be seen, cyclones are generally of smaller spatial scales and also involve much stronger pressure gradients, which also indicates stronger winds.

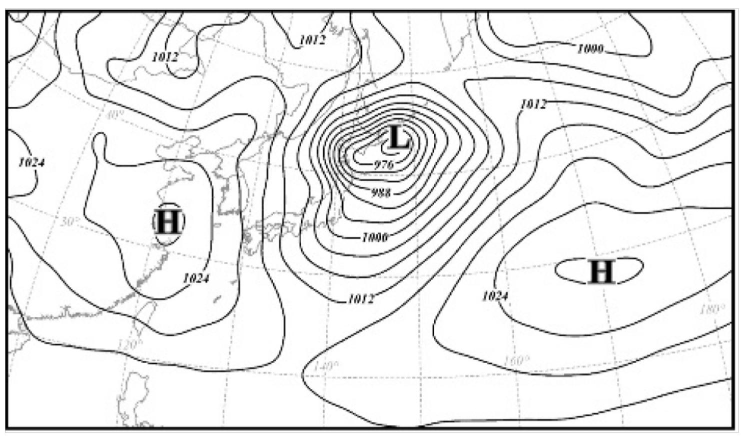

Figure 1: Sea-lavel pressure analysis for 0000 UTC 23 February 2004. Solid lines are isobars labeled in hPa and contoured every 4 hPa. Capital L and H represent centers of sea-level low- and high-pressure systems, respectively. Note the tight pressure gradient around the low and the much weaker pressure gradient around the highs. Figure taken from "Mid-Latitude Atmospheric Dynamics: A First Course" book, by Jonathan E. Martin (Figure 4.20, page 107)

To understand this, we next examine the force balance in each case.

Cyclones

For cyclones in the NH (Fig.2a), anti-clockwise circulation implies that the Coriolis force is acting outward, while the low pressure of the cyclones implies that the pressure gradient force is inwards (toward the low). The centrifugal force is outward in both cases.

The gradient wind balance is given by the equation:

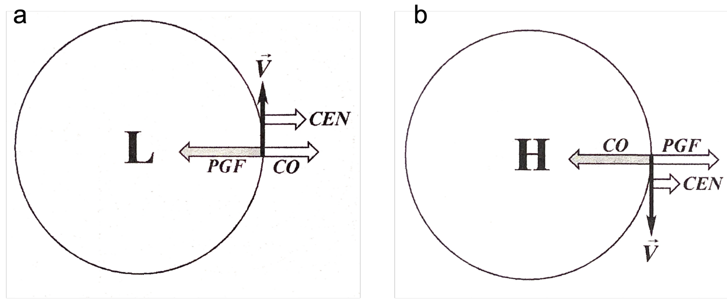

Figure 2: Force balance for a regular low 'L' (a) and regular high 'H' (b) in the Northern Hemisphere. The pressure gradient, Coriolis, and centrifugal forces are represented by PGF, CO, and CEN, respectively. Figure taken from "Mid-Latitude Atmospheric Dynamics: A First Course" book, by Jonathan E. Martin (Figure 4.17 and 4.19 on pages 104 and 106, respectively.)

$$f v_\theta + \frac{v_\theta^2}{r} = g\frac{dh}{dr} \tag{1}$$

where f is the Coriolis parameter, v_θ is the azimuthal wind speed, g is the acceleration due to gravity, h is the geopotential height, $\frac{dh}{dr}$ is the geopotential gradient, and r is the radial distance from the cyclone's center.

For cyclones, v_θ and $g\frac{dh}{dr}$ are both positive, since the rotation is anticlockwise (i.e., θ is increasing so $v_\theta = \frac{\partial\theta}{\partial t} > 0$), and the geopotential height (or similarly the pressure) is increasing outward radially.

Noting that $fv_g = g\frac{dh}{dr}$ for the geostrophic wind and plugging in (1), we find

$$\frac{v_\theta^2}{r} = f(v_g - v_\theta). \tag{2}$$

Now, since $\frac{v_\theta^2}{r} > 0$, it implies that $v_g > v_\theta$ for cyclones, meaning that the velocity is *subgeostrophic* i.e., smaller than the geostrophic wind.

Moreover, we can notice that (1) leads to a quadratic equation for v_θ,

$$\frac{v_\theta^2}{r} + f v_\theta - g\frac{dh}{dr} = 0,$$

whose solutions are given by the quadratic formula:

$$v_\theta = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

where $a = \frac{1}{r}$, $b = f$, and $c = -g\frac{dh}{dr}$.

Hence

$$v_\theta = \frac{-f \pm \sqrt{f^2 + \frac{4}{r} g\frac{dh}{dr}}}{\frac{2}{r}} = \frac{-rf}{2} \pm \sqrt{\frac{r^2 f^2}{4} + rg\frac{dh}{dr}}. \tag{3}$$

Since v_θ is positive for cyclones, the negative solution can be discarded and the overall solution is:

$$v_\theta = \frac{-rf}{2} + \sqrt{\frac{r^2 f^2}{4} + rg\frac{dh}{dr}}. \tag{4}$$

Note that an additional possible mathematical solution exists where the low pressure system is rotating clockwise (as opposed to anti-clockwise), which is coined an "anomalous cyclone", but these are rarely observed in nature.

Lastly, note also that (2) implies

$$\frac{v_g}{v_\theta} = \frac{v_\theta}{fr} + 1 = R_0 + 1, \tag{5}$$

which implies that the Rossby number can be estimated using the geostrophic-to-total wind ratio.

Anticyclones

For anticyclones in the NH (Fig.2b), clockwise circulation implies that Coriolis force is acting inward, while the high pressure of the anticyclones implies that the pressure gradient force is outward (away from the high). The centrifugal force is again outward as before.

In this case, v_θ and $g\frac{dh}{dr}$ are both negative, since the rotation is clockwise (i.e., θ is decreasing so $v_\theta = \frac{\partial\theta}{\partial t} < 0$), and the geopotential height (or similarly the pressure) is decreasing outward radially. Hence, the Coriolis term and the pressure gradient terms in (1) are now both negative, while the centrifugal term $\frac{v_\theta^2}{r}$ is still positive.

In this case, (2) can be rewritten as

$$\frac{v_\theta^2}{r} = f(v_g - v_\theta) = f(|v_\theta| - |v_g|), \tag{6}$$

and since $\frac{v_\theta^2}{r} > 0$, it must be that $|v_\theta| > |v_g|$ for anticyclones, meaning that the anticyclonic wind is *supergeostrophic* i.e., larger than the geostrophic wind.

The solutions to the quadratic equation for v_θ are now

$$v_\theta = \frac{-f \pm \sqrt{f^2 - \frac{4}{r}g|\frac{dh}{dr}|}}{\frac{2}{r}} = \frac{-rf}{2} \pm \sqrt{\frac{r^2 f^2}{4} - rg|\frac{dh}{dr}|}. \tag{7}$$

In this case, we notice that a solution exists only if $\frac{r^2 f^2}{4} - rg|\frac{dh}{dr}| > 0$, which implies that $g|\frac{dh}{dr}| < \frac{rf^2}{4}$. This suggests that the pressure gradient of an anticyclone is bounded, and approaches zero near the center of the anticyclone. No similar constraint exists for low pressure systems, and this remarkable difference is clearly seen in sea-level pressure charts (e.g., as in Fig.1). The weak pressure gradient near the center of the anticyclone also dictates that the winds will be weak in its vicinity, as opposed to the strong cyclonic winds. [1]

We again find that one of the solutions is unphysical and can therefore discarded, and the only physical solution is

$$v_\theta = \frac{-rf}{2} - \sqrt{\frac{r^2 f^2}{4} - rg|\frac{dh}{dr}|}. \tag{8}$$

Lastly, we also note that (6) implies in this case

$$\frac{|v_g|}{|v_\theta|} = 1 - \frac{|v_\theta|}{fr} = 1 - R_0. \tag{9}$$

Examining the gradient wind balance for real cyclones and anticyclones

We will use the Synoptic Laboratory website to plot v, v_g for real atmospheric data. Check last week- February 16th, around Washington (WA state) for a nice example.

Go to `http://synoptic.mit.edu/custom-plots/anyscalarwind/` and set:

1. In the "Scaler" field, change "tmpc" to "hght"
2. Set the day to "16" instead of today
3. In the "Wind-skip" option, change to yes (to reduce the number if arrows)
4. In the GAREA option, change "usnps" to "WA–".

This will produce a map with wind barbs and the 500mb geopotential height. Now repeat, but in the "Wind" option change "observed" to "Geostrophic".

Questions

1. Is the actual v smaller or larger than the geostrophic wind in the low and high region?
2. Can you estimate the Rossby number for each case?
3. Are winds generally stronger around the high or the low pressure?

[1] Note that the fact that $v_\theta < v_g$ for cyclones and $|v_\theta| > |v_g|$ for anticyclones does not imply that the anticyclone winds are stronger. This is true only for the *same pressure gradient*, which, as shown here, is not the case.