Global Energy Balance and the Greenhouse Effect

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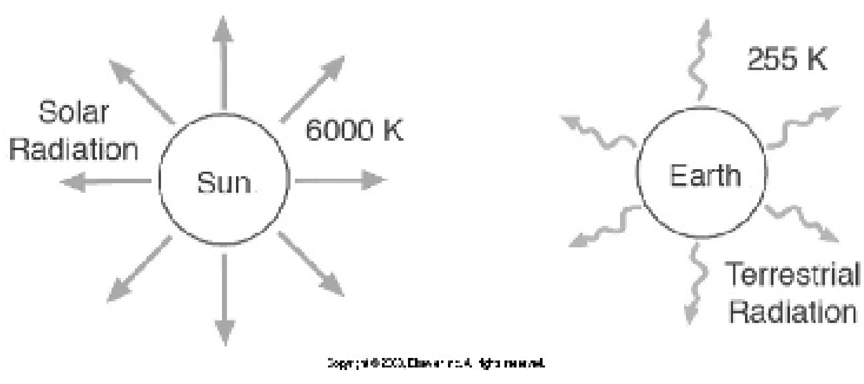
1 Global energy balance

Let us start by considering the Earth bathed in light from the Sun — see Fig.1 — and ask the question:

- What is the gross temperature of Earth?
- On what does that temperature depend?

Herev we learn that:

- $T_{SUN} = 6000K$ and it emits 'solar radiation' primarily in the visible.
- $T_{Earth} = 255K$ and it emits 'terrestrial radiation' primarily in the infra-red (IR).
- $T_{surface} = 288K > T_{Earth}$, a consequence of the 'Greenhouse effect'.



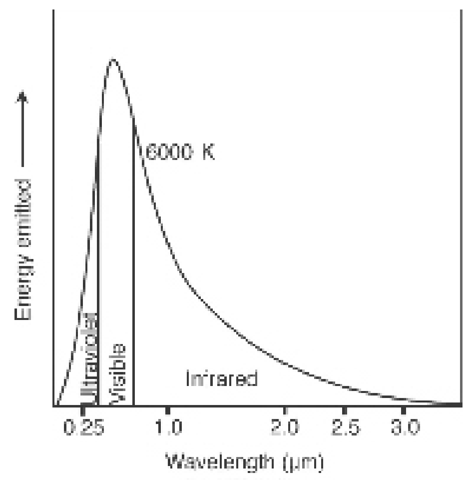


Figure 1: The energy emitted from the sun plotted against wavelength based on a black body curve with $T = T_{Sun}$. Most of the energy is in the visible and 95% of the total energy lies between 0.25 and 2.5 μm (10^{-6}m).

1.1 Emission temperature of Earth

Earth receives almost all its energy from Sun — only a small amount of geothermal heating.

Solar flux at the Earth is called the 'Solar constant', S_o — the INTENSITY of the radiation in which the Earth is bathed is:

$$S_o = 1367 \; Wm^{-2}$$

S_o depends on distance of the planet from the Sun. Not really constant because of variations in Earths orbit (e.g Milankovitch cycles)

The way in which radiation interacts with the atmosphere also depends on WAVELENGTH as well as the intensity. Relation between flux and wavelength is known as the spectrum.

The spectrum of solar radiation is shown in Fig.1. It peaks in the visible at a wavelength of $\lambda = 0.6\,\mu$m and decreases as λ increases and decreases. Note the colors of the rainbow — V, I, B, G, Y, O, R.

$$1 \text{ micron} = \mu\text{m} = 10^{-6} m$$

95% of all the energy lies between 0.25 and 2.5 μm — in the visible.

Why does spectrum have this pattern?

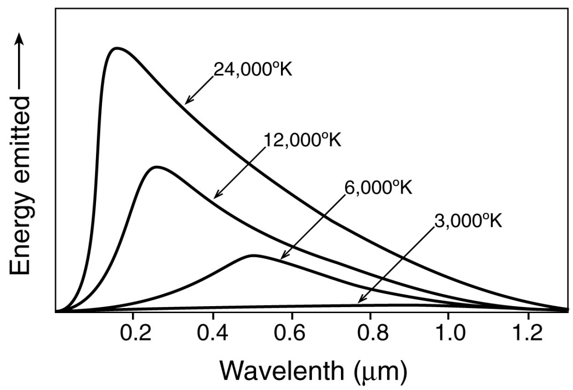


Figure 2: The energy emitted at different wavelengths for black bodies at several temperatures.

Such behavior is characteristic of the radiation emitted by *incandescent* material, as can be observed, for example, in a coal fire:

- The hottest parts of the fire are almost white and emit the most intense radiation with a wavelength that is shorter than that coming from the warm parts of the fire that glow red.
- The coldest parts of the fire do not seem to be radiating at all, but are, in fact, radiating in the infrared.

Experiment and theory show that the wavelength at which the intensity of radiation is a maximum, and the flux of emitted radiation depend only on the temperature of the source. The theoretical spectrum was worked out by Planck, and is known as the 'Planck' or 'blackbody'[1] spectrum.

It is plotted as a function of temperature in Fig.2.

If the observed radiation spectrum of the Sun is fitted to the black body curve, we deduce that the blackbody temperature of the sun is:

$$T_{SUN} = 6000K$$

Consider Fig.3.

[1] A black body is a theoretical construct that absorbs 100% of the radiation that hits it. Therefore it reflects no radiation and appears perfectly black. It is also a perfect emitter of radiation.

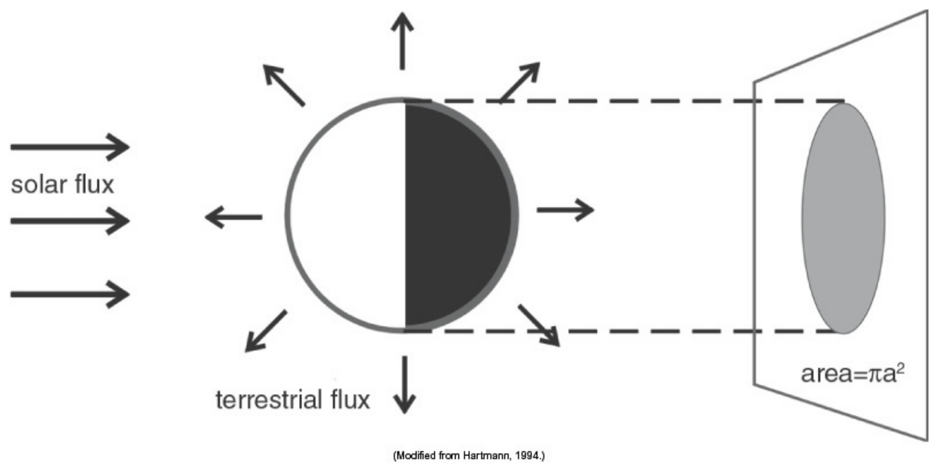


Figure 3: The spinning Earth is imagined to intercept solar flux over a disk and radiate terrestrial energy away isotropically from the sphere. Modified from Hartmann, 1994.

$$\text{Solar flux intercepted by Earth} = S_o \pi a^2$$

Not all radiation is absorbed — a significant fraction is reflected.

$$\alpha = \frac{\text{reflected radiation}}{\text{incident radiation}}$$

α is the albedo and depends on the nature of the reflecting surface — see Table1.
α large for snow, ice, cloud, desert; α low for ocean.
A map of the surface albedo is shown in Fig.4
Earth as a whole has an albedo $\alpha_p = 0.3$.

$$\text{Absorbed radiation} = (1 - \alpha_p) S_o \pi a^2$$

What about emitted radiation?

If Earth radiates according to the Planck law then:

$$\text{emitted radiation per unit area} = \sigma T_e^4$$

where σ = Stefan-Boltzman constant ($\sigma = 5.67 \times 10^{-8} \text{Wm}^{-2}\text{K}^{-4}$) and T_e is the emission (also called 'effective') temperature of the Earth.

Type of surface	Albedo
	(%)
Ocean	2 – 10
Forest	6 – 18
Cities	14 – 18
Grass	7 – 25
Soil	10 – 20
Grassland	16 – 20
Desert (sand)	35 – 45
Ice	20 – 70
Cloud (thin, thick stratus)	30, 60 – 70
Snow (old)	40 – 60
Snow (fresh)	75 – 95

Table 1: Albedos for different surfaces. Note that the albedo of clouds is highly variable and depend on the type and form.

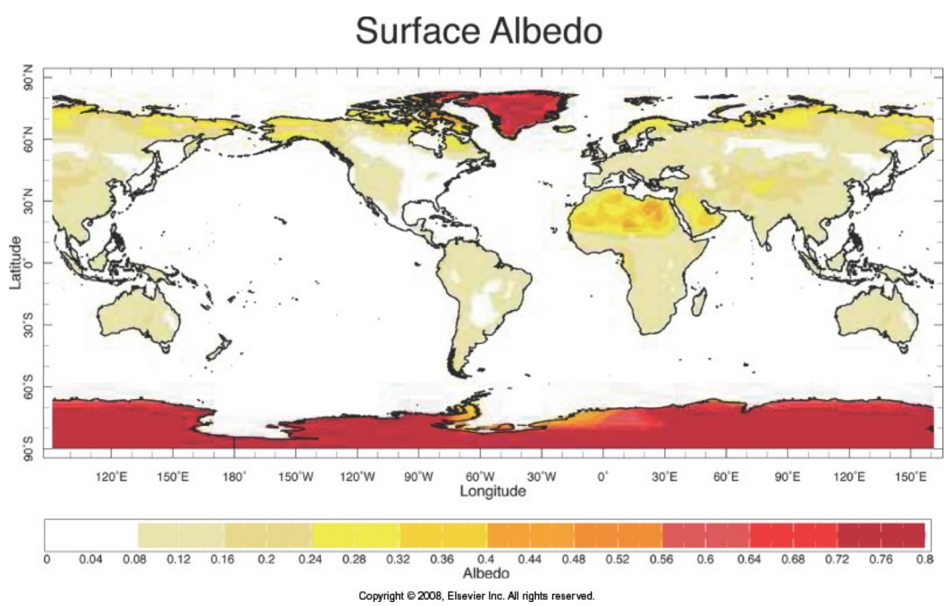


Figure 4: The albedo of the Earth's surface.

$$\text{Emitted radiation} = \sigma T_e^4 \times 4\pi a^2$$

because the Earth is rapidly rotating.

Equating emitted to absorbed we find that:

$$T_e = \left[\frac{S_o(1-\alpha_p)}{4\sigma} \right]^{1/4} \tag{1}$$

Note 'a', the radius of the Earth, does not appear.

T_e depends only on α_p and S_o. Putting in numbers for the Earth we find that:

$$T_e = 255K.$$

Note also that T_s (temperature at the surface) $\neq T_e$.

1.2 Atmospheric absorption spectrum

Property of Planck radiation curve is

$$\lambda_m T = \text{constant}$$

where λ_m is the wavelength at which the Planck curve peaks — see Fig.2.

Given that $\lambda_{m_{SUN}} = 0.6\,\mu\text{m}$, $T_{SUN} = 6000K$ and $T_{Earth} = 255K$, then λ_m for the Earth is

$$\lambda_{m_{Earth}} = \frac{6000}{255} \times 0.6\,\mu\text{m} = 14\,\mu\text{m}$$

which is in the far IR.

Thus Earth radiates back out to space in the far IR.

From Fig.5 we see that the black body spectra of the Sun and the Earth hardly overlap — this greatly simplifies our thinking about radiative transfer.

It turns out the atmosphere is largely transparent in the visible, but very opaque in the IR.

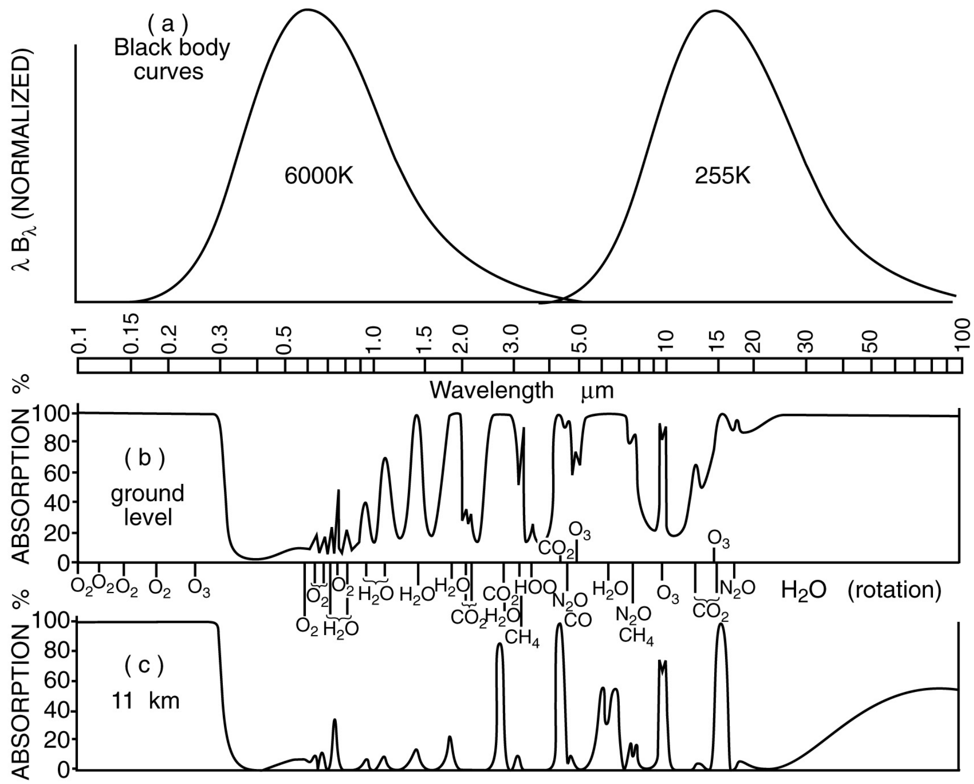


Figure 5: The normalized blackbody emission spectra,$T^{-4}\lambda B_\lambda$, for the Sun (6000K) and Earth (255K) as a function of $ln\lambda$(top) where B_λ is the black body function and λ is the wavelength. The fraction of radiation absorbed while passing from the surface to the top of the atmosphere as a function of wavelength (middle). The fraction of radiation absorbed from the tropopause (typically at a height of 11km) to the top of the atmosphere as a function of wavelength (bottom). The atmospheric molecules contributing the important absorption features at each frequency are also indicated: after Goody and Yung: "Atmospheric Radiation", Oxford Univ. Press, 1989.

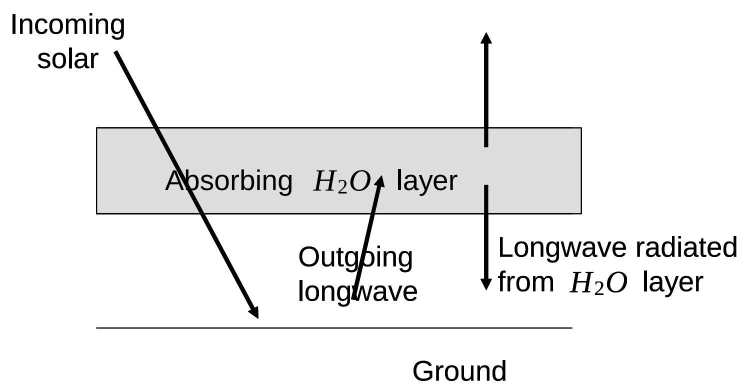


Figure 6: Schematic of 'greenhouse effect'.

2 The Greenhouse effect

The emission temperature is the temperature that we would deduce if we looked back at the Earth from space, measured the terrestrial radiation coming from it and fitted a black body curve.

But the observed surface temperature of the Earth is:

$$T_s = 288K$$

and is considerably higher than T_e. The mechanism by which T_s becomes raised relative to T_e has became known as Greenhouse effect.

The essential process at work in the Greenhouse effect is sketched in Fig.6. The atmosphere is fairly transparent to incoming solar radiation (but remember that a fraction α gets reflected back to space). But the Earth radiates primarily in the IR and it turns out that the atmosphere is strongly absorbing in IR due to presence of triatomic molecules — primarily H_2O, CO_2, O_3. By far the most important of these is H_2O. The water vapor layer enveloping the Earth absorbs much of the IR upwelling from the surface and re-radiates it out to space, but also back down to the surface. Thus the radiation from the earth's surface — see Fig.6 — has to balance not just the incoming solar radiation but also the downward longwave radiation from the H_2O layer. It thus has to raise its temperature above that of the atmosphere. This has become known as the Greenhouse effect.

Let's begin by looking at an atmospheric absorption spectrum — the fraction of the radiation at each λ absorbed in a single vertical path through the atmosphere dominated by triatomic molecules (H_2O, CO_2 + others in IR; O_3 in UV) — shown in Fig.5.

From it we see:

- the atmosphere is almost completely transparent in the visible, at the peak of the solar spectrum.
- the atmosphere is very opaque in the UV.
- the atmosphere is fairly opaque across the IR spectrum—almost completely opaque at some wavelengths, transparent at others.
- although air ≡ mixture of gases in (almost) constant ratio $(N_2, O_2) \sim (80\%, 20\%)$ (see Table 2) N_2 does not figure at all in absorption, and O_2 absorbs only in the far UV (where there is little solar flux) and, a little, in the near IR: the dominant constituents of the atmosphere are incredibly transparent across almost the whole spectral range of importance.
- the absorption is dominated by triatomic molecules — O_3 in the UV, H_2O, CO_2 and others in the IR because it so happens that triatomic molecules have rotational and vibrational modes that can easily be excited by radiation with wavelengths in the IR. These triatomic molecules are present in tiny concentrations — see Table 2. This is the basic reason why atmospheric radiation is so vulnerable to human-induced changes in composition.

2.1 A simple greenhouse model

Average incoming solar flux $= \frac{\text{intercepted incoming radiation}}{\text{Earth's surface area}} = \frac{S_o \pi a^2}{4\pi a^2} = \frac{S_o}{4}$.

As in Fig.7, we represent the atmosphere by a single layer of temperature T_a and, in this first calculation, assume that the atmosphere is:

1. completely transparent to shortwave (visible) radiation
2. completely opaque in IR.

chemical species	molecular weight ($g\ mol^{-1}$)	proportion by volume		chemical species	molecular weight	proportion by volume
N_2	28.01	78%		O_3	48.00	∼500ppb
O_2	32.00	21%		N_2O	44.01	310ppb
Ar	39.95	0.93%		CO	28.01	120ppb
H_2O (vapor)	18.02	∼0.5%		NH_3	17.03	∼100ppb
CO_2	44.01	360ppm		NO_2	46.00	∼1ppb
Ne	20.18	19ppm		CCl_2F_2	120.91	480ppt
He	4.00	5.2ppm		CCl_3F	137.37	280ppt
CH_4	16.04	1.7ppm		SO_2	64.06	∼200ppt
Kr	83.8	1.1ppm		H_2S	34.08	∼200ppt
H_2	2.02	∼500ppb		AIR	28.97	

Table 2: The most important atmospheric constituents. The chlorofluorocarbons (CFCs) CCl_2F_2 and CCl_3F are also known as CFC-12 and CFC-11 respectively. [N.B. (ppm, ppb, ppt) = parts per (million, billion, trillion)]

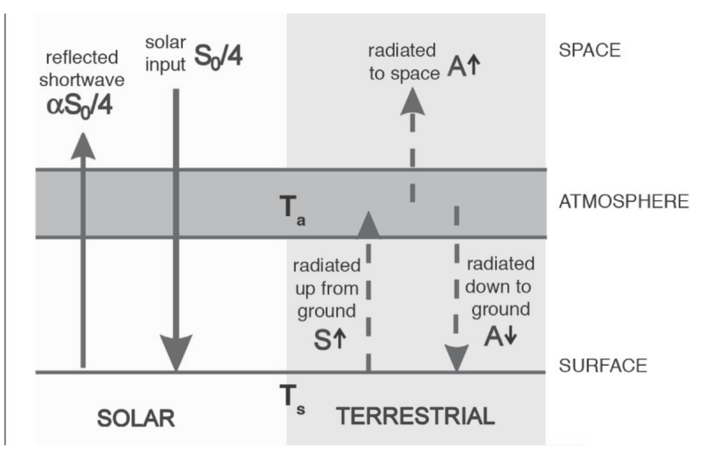


Figure 7: The simplest greenhouse model, comprising a surface at temperature T_s, and an atmospheric layer at temperature T_a, subject to incoming solar radiation $\frac{S_o}{4}$. The terrestrial radiation upwelling from the ground is assumed to be completely absorbed by the atmospheric layer.

Now consider the balance of radiation in Fig.7.

(i) Balance at top of atmosphere

$$(1-\alpha_p)\frac{S_o}{4} = A\uparrow = \sigma T_a^4$$

assuming the atmosphere radiates as a blackbody. Thus

$$T_a = T_e = \left[\frac{(1-\alpha_p)\,S_o}{4\sigma}\right]^{\frac{1}{4}}$$

We see that atmosphere is at the emission temperature — naturally because it's the region emitting to space.

(ii) Balance at the ground

$$(1-\alpha_p)\frac{S_o}{4} + A\downarrow = \sigma T_s^4$$

but

$$A\uparrow = A\downarrow = (1-\alpha_p)\frac{S_o}{4}$$

Thus

$$2\left[\frac{(1-\alpha_p)\,S_o}{4}\right] = \sigma T_s^4$$

and so

$$T_s = 2^{\frac{1}{4}} T_a$$

$2^{\frac{1}{4}} = 1.1892$ and so, if $T_a = 255$, we find that $T_s = 303K$.

Note:

$$T_s > T_a = T_e$$

:the surface is hotter than the atmospheric layer.

The atmospheric temperature is $T_a = T_e$ — outgoing radiation emanates not from the surface but from the atmosphere above.

[Question: How does a real greenhouse work?[2]]

[2] It is interesting to note that the domestic greenhouse does not work in this manner! A greenhouse made of plastic window panes, rather than conventional glass, is effective even though plastic (unlike glass) does not have significant absorption bands in the IR. The greenhouse works because its windows allow energy in and its walls prevent the warm air from blowing away.

The predition of this simple model is closer to the observed surface temperature, but overestimates it somewhat.

Model is too simple because

(i) not all of the solar flux incident at the top of atmosphere reaches ground — some gets absorbed.

(ii) IR absorption is incomplete in the real atmosphere — the Greenhouse effect is not as strong as represented in this model.