Notes on Potential Temperature

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1 Compressible versus incompressible fluid

Water is an incompressible fluid whose density depends mainly function on temperature (here we neglect the dependence on salinity): $\rho = \rho(T)$. In adiabatic conditions, the temperature of a water parcel is conserved as it moves around and thus temperature can be used as tracer.

Air, instead, is a compressible fluid whose density depends on both temperature and pressure: $\rho = \rho(p, T)$. Indeed the atmosphere closely obeys the perfect gas law, $\rho = p/RT$, where R is the gas constant for dry air $R = 287JK^{-1}Kg^{-1}$. As a parcel of air rises, for example, it moves into an environment of lower pressure. The parcel will adjust to this pressure; in doing so it will expand, do work on its surroundings, and thus cool. So in the atmosphere the parcel temperature is not conserved during displacement, even if that displacement occurs adiabatically.[1] Temperature cannot be used as a tracer.

1.1 The dry adiabatic lapse rate

Imagine, then, that a parcel of air is displaced vertically. How much does it cool?

Using the first law of thermodynamic and the perfect gas law, it can be shown that the rate at which the temperature decreases with height under adiabatic displacement is:

$$\text{the dry adiabatic lapse rate } \Gamma_d = -g/c_p \sim 10^o K/\text{kilometer}$$

[1]Here will have assumed a dry atmosphere. Moisture modifies things because as air cools water vapor can condense releasing latent heat and warming the parcel.

Think about climbing a mountain. As you ascend the temperature gets colder and colder. For example on going up Mt. Washington, which is ~2 km high, on top of the mountain the temperature will be much lower than at sea-level. On a summer day the temperature in Boston could be ~30^oC while it might be close to 10^oC on Mt.Washington!

2 Potential temperature

The non-conservation of T under adiabatic displacement makes it a less than ideal measure of atmospheric thermodynamics. However we can define a temperature called *potential temperature* which is conserved in adiabatic displacement and so can be used as a tracer. The potential temperature of an air parcel, denoted by θ, is the temperature it would have if it were compressed adiabatically from its existing p and T to a standard pressure.

The definition of potential temperature is:

$$\theta = T\left(\frac{p_0}{p}\right)^{\kappa} \tag{1}$$

with $k = R/c_p = 9/7$ and conventionally $p_o = 1000mb$.

In adiabatic conditions, it follows that:

$$\frac{d\theta}{\theta} = \frac{dT}{T} - k\frac{dp}{p} = 0 \tag{2}$$

Unlike T, θ is conserved in a compressible fluid under (dry) adiabatic conditions. Parcels of air can be labelled by their potential temperature θ.

Fig 4.9 (from the Marshall and Plumb book) shows that climatologically atmosphere temperature decreases with height, with a lapse rate close to the dry adiabatic lapse rate of 10^oK/kilometer until the tropopause level. The potential temperature instead increases with height until the tropopause, and then increases more rapidly in the stratosphere.

For a more complete derivation of potential temperature see Chapter 4 of Marshall and Plumb book.

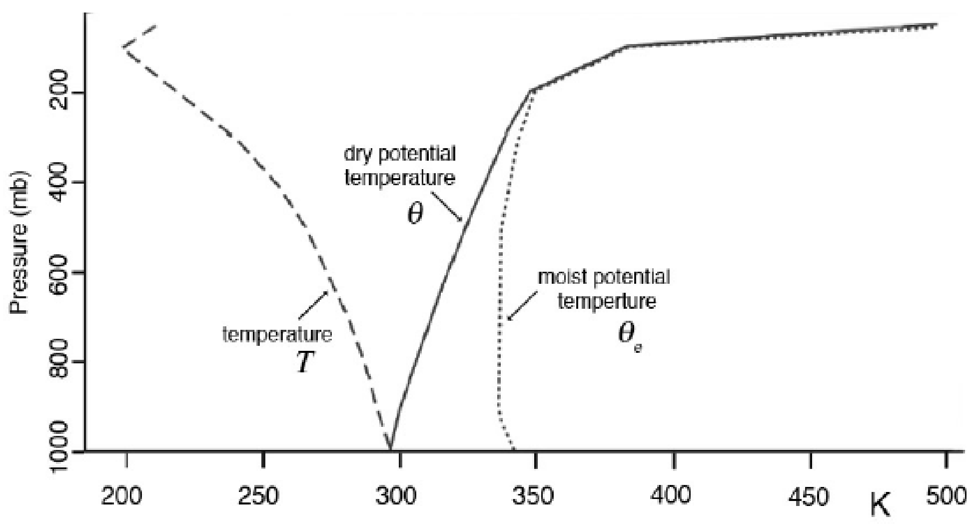

Figure 4.9: Climatological atmospheric temperature T (dashed), potential temperature θ (solid) and equivalent potential temperature θ_e (dotted) as a function of pressure, averaged over the tropical belt $\pm 30°$.

From its definition (1), we can see that θ is the temperature a parcel of air would have if it were expanded or compressed adiabatically from its existing p and T to a standard pressure p_0. It allows one, for example, to directly determine how the temperature of an air parcel will change as it is moved around adiabatically: if we know its θ, all we need to know at any instant is its pressure, and then (1) allows us to determine its temperature at that instant.

For example, using the climatological profile of Fig 4.9, a parcel of air at $300mb$ has $T = 229K$ (which is -44^oC), and $\theta = 323K$. If the parcel was brought down adiabatically, thus conserving θ, to the ground ($p = p_0$), its temperature will be: $T = \theta = 229 \left(\frac{1000}{300}\right)^{9/7} = 323K$ (or 50^oC).

3 Example: Polar Front on 130117 12z

Here we are discussing two N-S sections through the polar front, discussed in Part 3 of assignment 3 on Fronts. The first is a section of temperature T in ^oC. The second is a section of potential θ temperature in ^oK.

Note that T decreases markedly with height as would be expected from the dry adiabatic lapse rate. The front is not easy to locate: the boundary between the cold and warm air is located where the isotherms are nearly vertical. On the other hand, potential temperature the front is easily located from the potential temperature field and is marked, roughly, by he 295^oK contour.

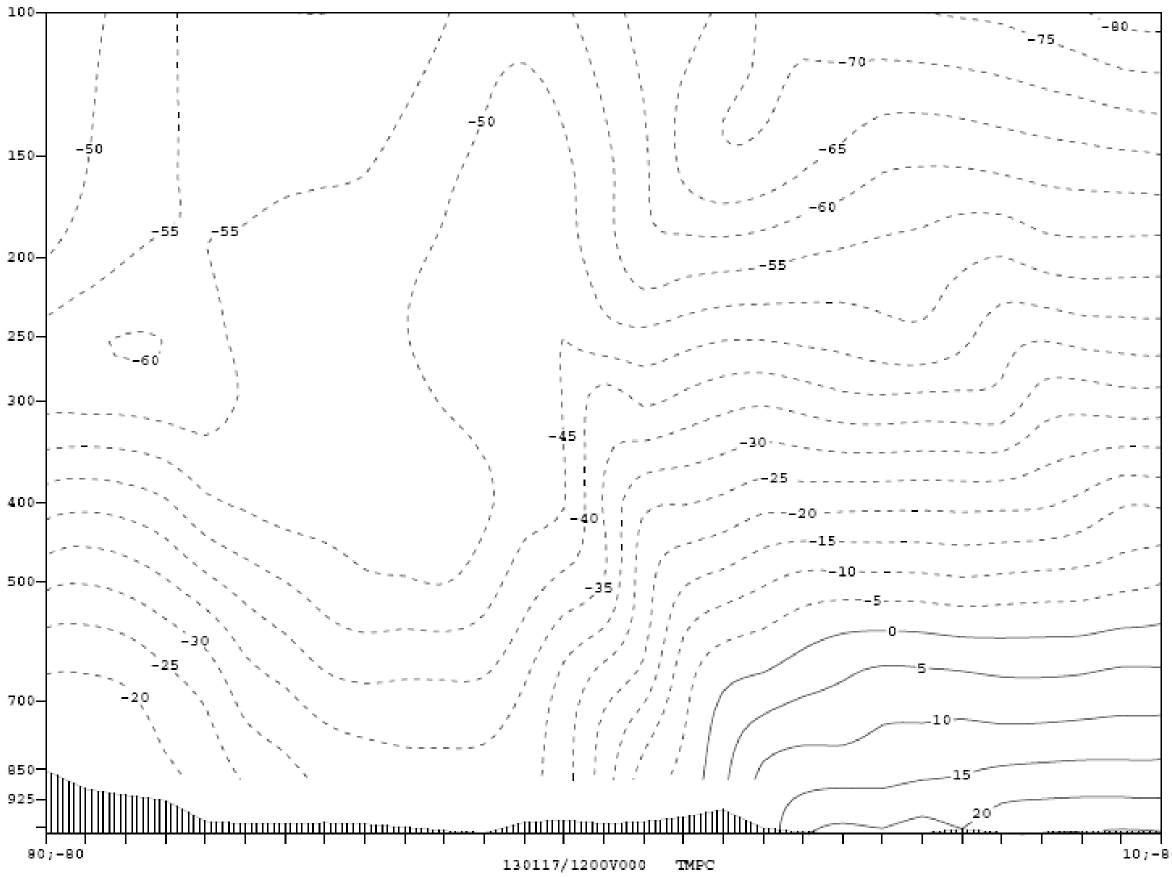

Figure 1: N-S section of temperature in ^oC along the 80^oW meridian. Vertical axis is $\log p$ (proportional to height). Horizontal axis is latitude (from 80^oN to 10^oN).

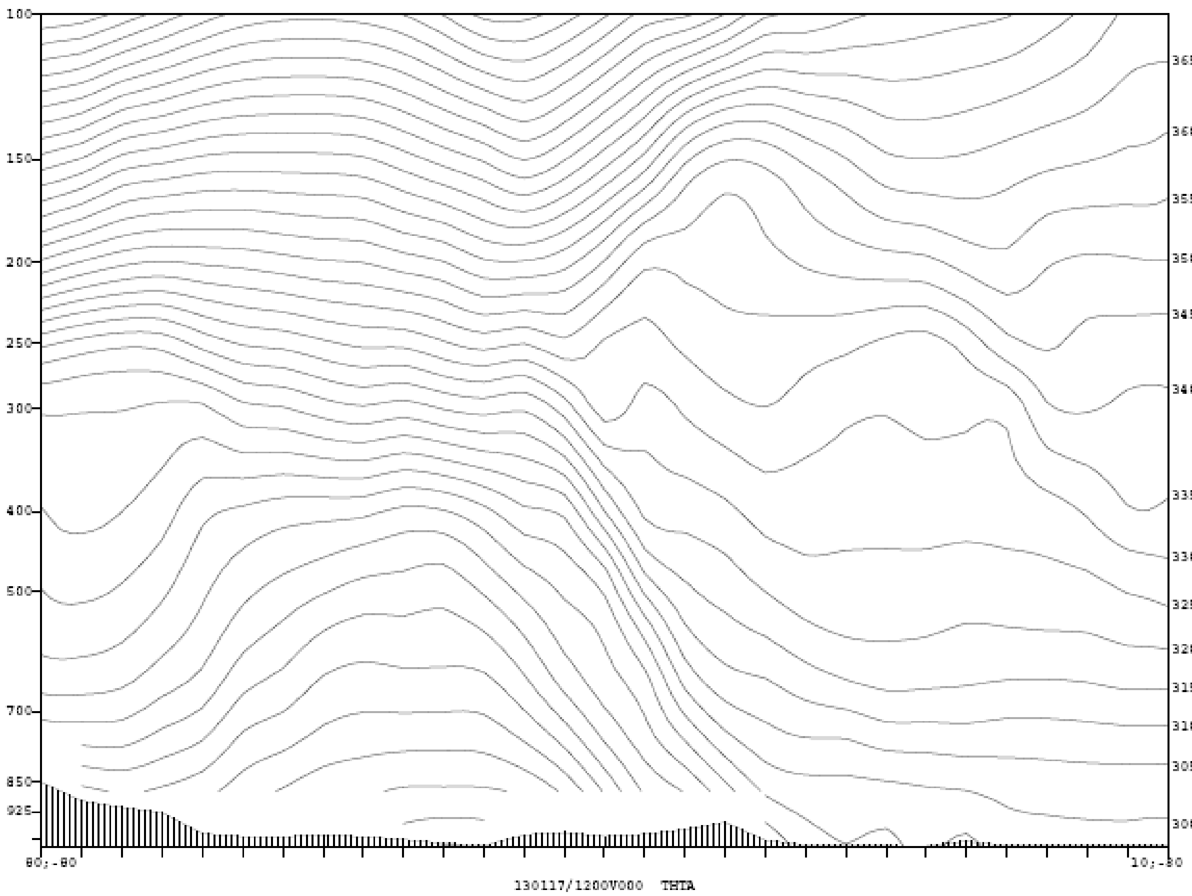

Figure 2: N-S section of potential temperature in ^oK along the 80 W meridian. Vertical axis is $\log p$ (proportional to height). Horizontal axis is latitude (from 80^oN to 10^oN)