12.307: Project 2
Tracer Transport in the Atmosphere and Ocean
Atmospheric Aerosols and Oceanic Garbage Patches

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1 Introduction

In Project 1 we learned that the large scale flow of the atmosphere (and indeed ocean) is close to geostrophic (i.e the Rossby number is small with Coriolis forces balancing pressure gradient forces) and strongly influenced by Earth's rotation. In this second project — Project 2 — we study how winds and currents carry with them their properties as they move around the globe. We have already seen a very vivid example of fluid transport in our introductory lecture when we stirred a dye in to a rotating fluid — see Fig.1. Water parcels, marked by the dye, fold around one-another creating beautiful patterns of inter-mingling colors.

In Project 2 we will explore how 'tracers' are transported by winds and currents in the Atmosphere and Ocean and how their dispersal depends on the scale of the motion and Earth's rotation. Examples of tracers are, as illustrated in Fig.2:

- Atmosphere:
  aerosols, dust from volcanoes or wild fires, radioactive plumes, water vapor......
- Ocean:
  plastics, oil spills, phytoplankton blooms.....

Our project will comprise two elements:

1. Fluid Laboratory: Exploring the dispersion of plastics in the ocean.

   We will use a laboratory analogue of the wind-driven ocean circulation to study how ocean currents carry and gather together discarded plastics in to particular regions of

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Figure 1: Taylor columns revealed by food coloring in the rotating tank. The water is allowed to come into solid body rotation and then gently stirred by hand. Dyes are used to visualize the flow. At the top we show the rotating cylinder of water with curtains of dye falling down from the surface. Below we show the beautiful patterns of dyes of different colors being stirred around one another by the rotationally constrained motion.

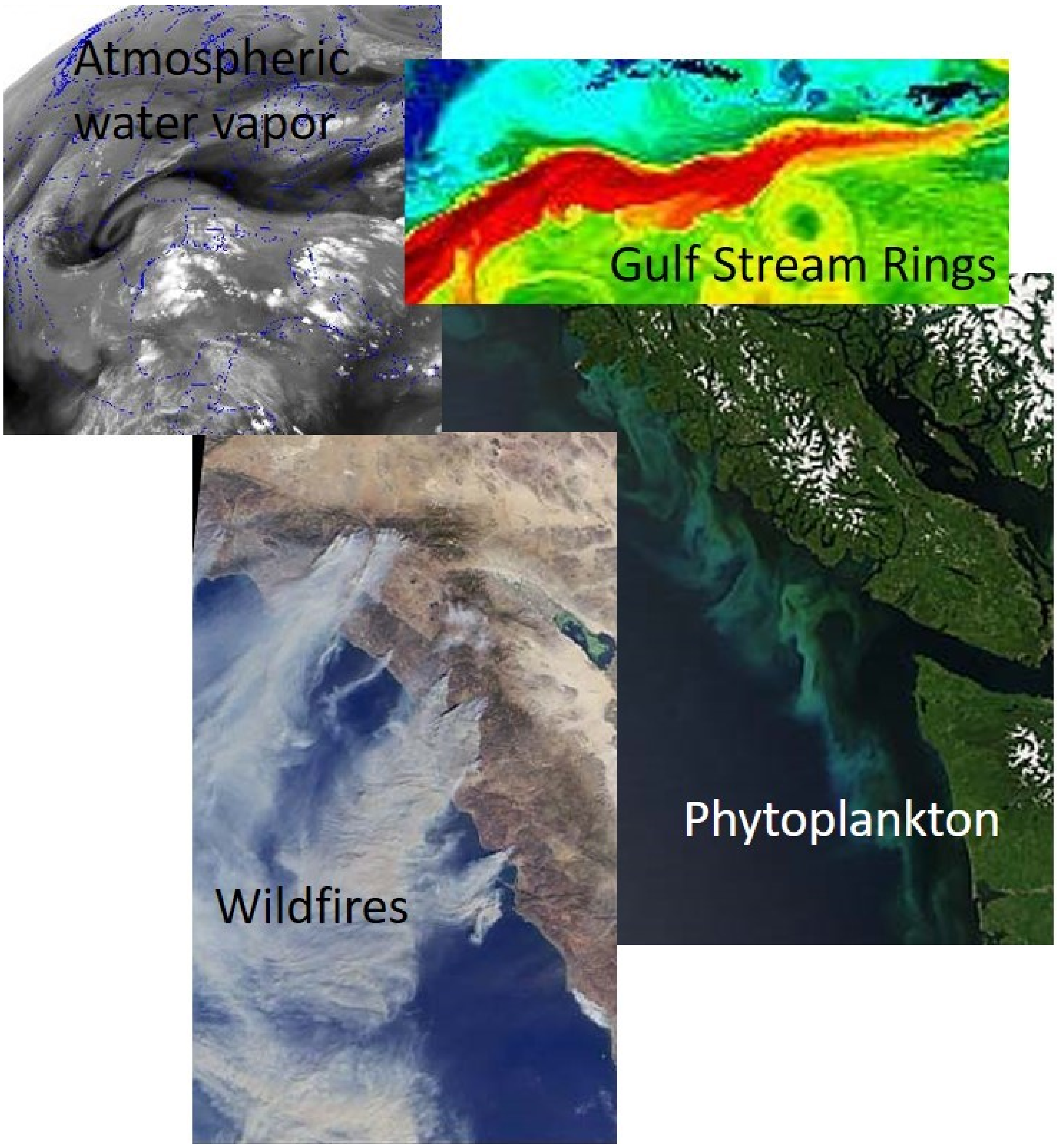

Figure 2: Winds advect tracers such as aerosols, dust from volcanoes or wild fires, radioactive plumes and water vapor, and many other natural and anthropogenic materials Ocean currents transport plastics, oil spills, phytoplankton blooms, etc. The rotation of the earth profoundly influences the patterns and character of these tracer fields, winding them in to wonderful, intricate patterns.

the ocean. In an associated activity we will release virtual particles in to a global ocean using the EsGlobe and explore where and how garbage patches — large floating areas of plastics — are created.

Plastic litter floating in the surface waters of the ocean follow ocean currents and trace them out, becoming concentrated in giant pools known as 'garbage patches'. They are also one of the biggest environmental challenges facing mankind. The plastics can entangle marine animals, they can mistake the plastic materials for food and eat it. Once eaten, plastics and the chemicals within them, can enter the food-chain with potentially harmful effects.

2. Tracer transport in the atmosphere.

   We will explore tracer transport in the atmosphere by tracking particles, estimating speeds and scales, using EsGlobe, including, for example: release of radioactive material (Fukushima), dispersal of aerosols, satellite water vapor, atmospheric patches of particles.

Before going on we introduce some important background theory which enables us to express the rate of change of a property of a fluid element, *following that element as it moves along*, rather than at a fixed point in space.

1.1 When is a fluid not a fluid? — Differentiation following the motion

Consider the situation sketched in Fig.3 in which a wind blows over a hill. The hill produces a pattern of waves in its lee. If the air is sufficiently saturated in water vapor, the vapor often condenses out to form cloud at the 'ridges' of the waves.

Let us suppose that a steady state is set up so the pattern of cloud does not change in time. If $C = C(x, y, z, t)$ is the cloud amount, where (x, y) are horizontal coordinates, z is the vertical coordinate, t is time, then:

$$\left(\frac{\partial C}{\partial t}\right)_{\substack{\text{fixed point}\\ \text{in space}}} = 0,$$

where we keep at a fixed point in space, but at which, because the air is moving, there are constantly changing fluid parcels. The derivative $\left(\frac{\partial}{\partial t}\right)_{\text{fixed point}}$ is called the 'Eulerian derivative' after Euler.

But C is not constant *following along a particular parcel*; as the parcel moves upwards into the ridges of the wave, it cools, water condenses out, cloud forms, and so C increases;

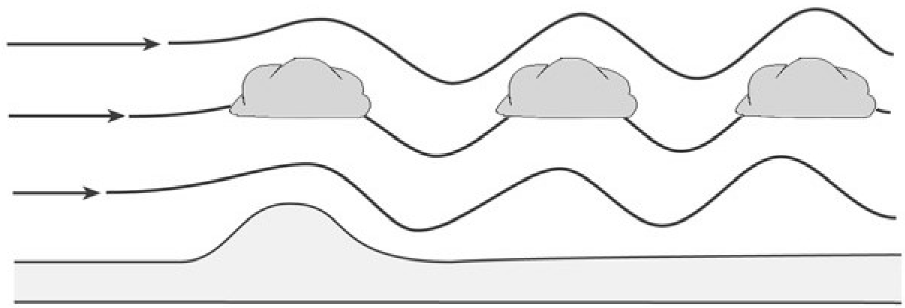

Figure 3: A schematic diagram illustrating the formation of mountain waves (also known as lee waves). The presence of the mountain disturbs the air flow and produces a train of downstream waves (cf., the analogous situation of water in a river flowing over a large submerged rock, producing a downstream surface wave train). Directly over the mountain, a distinct cloud type known as lenticular ("lens-like") cloud is frequently produced. Downstream and aloft, cloud bands may mark the parts of the wave train in which air has been uplifted (and thus cooled to saturation).

as the parcel moves down into the troughs it warms, the water goes back in to the gaseous phase, the cloud disappears and C decreases. Thus

$$\left(\frac{\partial C}{\partial t}\right)_{\substack{\text{fixed}\\ \text{particle}}} \neq 0$$

even though the wave-pattern is fixed in space and constant in time.

So, how do we mathematically express 'differentiation following the motion'? In order to follow particles in a continuum a special type of differentiation is required. Arbitrarily small variations of $C(x, y, z, t)$, a function of position and time, are given to the first order by:

$$\delta C = \frac{\partial C}{\partial t}\delta t + \frac{\partial C}{\partial x}\delta x + \frac{\partial C}{\partial y}\delta y + \frac{\partial C}{\partial z}\delta z$$

where the partial derivatives $\frac{\partial}{\partial t}$ etc. are understood to imply that the other variables are kept fixed during the differentiation. The fluid velocity is the rate of change of position of the fluid element, following that element along. The variation of a property C *following an element of fluid* is thus derived by setting $\delta x = u\delta t$, $\delta y = v\delta t$, $\delta z = w\delta t$, where u is the speed in the x-direction, v is the speed in the y-direction and w is the speed in the z-direction, thus:

$$(\delta C)_{\substack{\text{fixed}\\ \text{particle}}} = \left(\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} + w\frac{\partial C}{\partial z}\right)\delta t$$

where (u, v, w) is the velocity of the material element which by definition is the fluid velocity.

Dividing by δt and in the limit of small variations we see that:

$$\left(\frac{\partial C}{\partial t}\right)_{\substack{\text{fixed}\\ \text{particle}}} = \underbrace{\frac{\partial C}{\partial t}}_{\text{fixed point}} + \underbrace{u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} + w\frac{\partial C}{\partial z}}_{\text{advection}} = \frac{DC}{Dt} \tag{1}$$

in which we use the symbol $\frac{D}{Dt}$ to identify the rate of change following the motion:

$$\frac{D}{Dt} \equiv \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z} \equiv \frac{\partial}{\partial t} + \mathbf{u}.\nabla \ . \tag{2}$$

Here $\mathbf{u} = (u, v, w)$ is the velocity vector and $\nabla \equiv \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$ is the gradient operator. Note that the second term in Eq.(1) is known as 'advection' and represents the property of a fluid to carry proporties along with it as it moves.

The expression $\frac{D}{Dt}$ is called the Lagrangian derivative (after Lagrange; 1736 - 1813) [it is also called variously the 'substantial', the 'total' or the 'material' derivative]. Its physical meaning is 'time rate of change of some characteristic of a particular element of fluid' (which in general is changing its position). By contrast, as introduced above, the Eulerian derivative $\frac{\partial}{\partial t}$, expresses the rate of change of some characteristic at a *fixed point* in space (but with constantly changing fluid element because the fluid is moving).

Examples:

- Velocity and trajectories.

  The position of the parcel of fluid is related to its velocity thus (in 2-dimensions):

$$u = \frac{D}{Dt}x; \quad v = \frac{D}{Dt}y \tag{3}$$

$$x = \int u dt; \ y = \int v dt \tag{4}$$

  where u is the speed in the x direction and v is the speed in the y direction etc.

- Tracer transport.

  If a tracer concentration of a parcel of fluid, T say, does not change as it moves along, then:

$$\frac{D}{Dt}T = 0. \tag{5}$$

  This is clearly illustrated in Fig.1 where fluid parcels conserve (but for small diffusive processes) the concentration of dye that marked parcels when the dye was injected. Over time the fluid parcels intermingle drawing the colors they carry in to close proximity to form the beautiful swirling patterns.

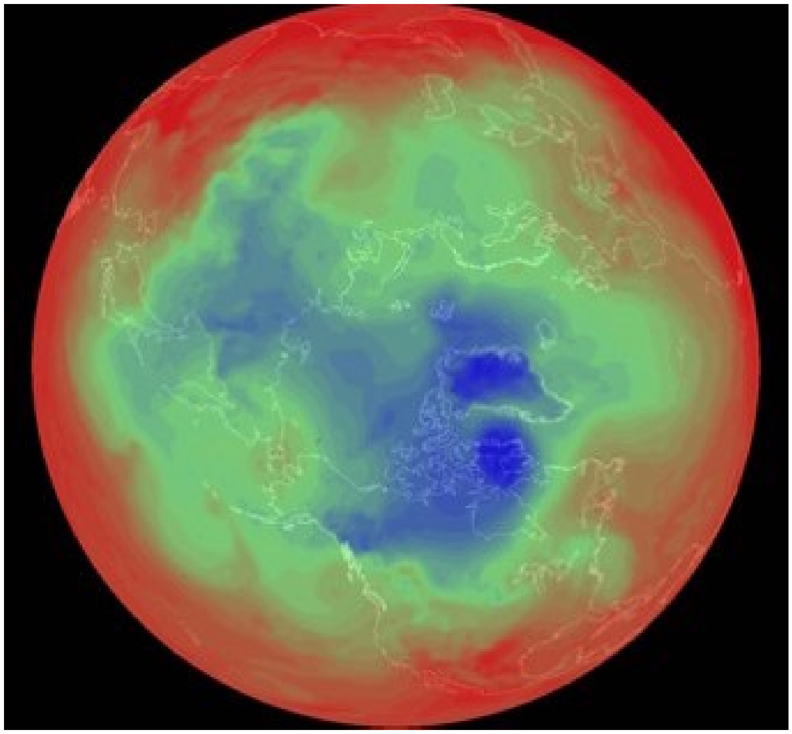

Figure 4: Instantaneous map of the northern hemisphere temperature at 850mb (roughly 2 km above the surface) on a day in January. Warm is red and cold is blue. You can view a movie loop here: http://weathertank.mit.edu/wp-content/uploads/2017/04/850mbPotTempLOOP.gif

- Familiar meteorological example.

  Figure 4 shows an instantaneous map of the northern hemisphere temperature at 850mb (roughly 2 km above the surface) on a day in January. The red shows warm air in the tropics, the blue cold air of polar latitudes. In regions where the cold (blue) air is moving south ($v < 0$), the local rate of change of temperature is[1]:

$$\frac{\partial T}{\partial t} \simeq -v\frac{\partial T}{\partial y} < 0$$

  because $v < 0$ and $\frac{\partial T}{\partial y} < 0$. Thus at a particular point on the Earth (e.g. Boston), the temperature decreases. The reverse happens when warm (red) air moves north. This is the process known as ‘advection’.

[1] Here y increases north and the meridional wind speed is $v = Dy/Dt$ and is positive if the wind is blowing north. The meridional temperature gradient is $\partial T/\partial y < 0$ if it gets colder moving north.